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# Numerical approximation of singular asymptotic states for a size-structured population model with a dynamical resource

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ABSTRACT

We compare two selection procedures in a numerical method proposed for the numerical simulation of the long time behaviour of solutions for a size-structured population model whose dependency on the environment is managed by the evolution of a vital resource and with a free sign growth rate. We obtain the approximation of singular asymptotic states such as stable steady ones and attractive limit cycles.



## 1. Introduction

Structured population models combine knowledge of individuals within the population (its basic unit) with notions of higher organisational levels. Their purpose is to describe the dependence of the dynamics of the whole population on the physiological state of the individuals, and viceversa, and are usually conceived as a frequency distribution of individuals which evolves over time. This aspect can be modelled by "structuring" the population with continuous, internal variables which represent a particular physiological feature. At the beginning of the last century, the most common variable used to model structured populations was age (see [1] for a detailed description of such models and numerical methods employed to solve them). However, this variable has limited practical value due to the fact that age is very difficult to measure directly in a large number of species. As an alternative, other measurable physiological characteristics (such as length, mass, stage of maturity, energy reserves, amount of foliage, etc.; which can be grouped under the umbrella term "size") have been used to model the dynamics. Such studies led to the formulation of size-structured population models (see [2–5] for a detailed description of such models with real applications and numerical methods employed to solve them). In this kind of models, the competition between individuals for resources is usually driven through quantities as the total population or the biomass. However, in the literature, this influence is sometimes determined by a dynamical resource as in the model of a *Daphnia magna* population [6]. More details about physiologically structured models can be found in [7–10].

In the following, we introduce a size-structured population model with a dynamical resource. It involves a nonlinear hyperbolic-partial differential equation (the population balance law)

$$u_t + (g(x, S(t), t)u)_x = -\mu(x, S(t), t)u, \quad 0 < x < x_M(t),\ t > 0, \tag{1.1}$$

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