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Biofilm growth on medical implants with randomness

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ABSTRACT

Biofilms are colonies of bacteria that attach to surfaces by producing extracellular polymer substances. They may cause serious infections in humans and animals, and also cause problems in hydraulic machinery. In this paper we model the growth of a biofilm established on a medical implant. We assume that the biofilm's growth is given by a logistic reaction term with the growth rate being a random variable with a given distribution. This way we take into account the variability in the bacterial populations, and the measurement and experimental errors. The diffusion coefficient of the microbes is also taken to be random. A stochastic spectral representation of the parameters and the unknown stochastic process is used, together with the polynomial chaos method, to obtain a system of partial differential equations, which is integrated numerically to obtain the evolution of the mean and higher-order moments with respect to time. Some examples are presented.

1. Introduction

Device-centered infections are devastating complications associated with implanted medical devices [1]. Even under the most aseptic conditions some bacteria will be present on the device at the time of the implant. If during the approximately first 18 h after the implant, the body's white cells and the antibiotic therapies have not succeeded in eliminating the bacteria, these will start attaching to the surface, producing extra-cellular polymers, and colonizing, forming biofilms. These are very difficult to combat, and then usually the only effective treatment is removal of the implant. There is a need to better understand the biofilm growth processes on medical devices. Mathematical models can help. One such model is described by reaction–diffusion partial differential equations which are obtained by averaging and perturbation techniques from conservation laws [2].

The objective of our research work is to develop a model of biofilm growth on a medical implant incorporating randomness. The growth of the microbes is given by a logistic growth model with competition for resources as the limiting growth factor. The microbes are attached to the surface of the implant and we assume that they can only move by diffusion. The model of the microbes is, therefore, given by a diffusion–reaction equation. The growth rate parameter as well as the diffusion coefficient will be considered random variables and the concentration of microbes will therefore be an stochastic process depending on position and time. Randomness in the growth rate coefficient may arise because of errors in the observed or measured microbial data, variability of the microbial populations, uncertainties like missing data, or plainly because of lack of knowledge. Lack of knowledge on how microbes move in a biofilm is the main reason to consider randomness in the diffusion coefficient.

In this paper we use the polynomial chaos approach [3,4] to study the time evolution of a partial differential equation for microbial biofilm growth in porous media. We will assume that competition is the limiting factor in the microbial growth,

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