Mathematical and Computer Modelling 54 (2011) 2515–2527

Contents lists available at ScienceDirect

Mathematical and Computer Modelling

journal homepage: www.elsevier.com/locate/mcm

A regularized smoothing Newton method for solving the symmetric cone complementarity problem☆

Changfeng Ma *

School of Mathematics and Computer Science, Fujian Normal University, Fuzhou 350007, China

ARTICLE INFO

Article history:
Received 27 May 2010
Received in revised form 7 June 2011
Accepted 7 June 2011

Keywords:
Symmetric cone complementarity problem
Euclidean Jordan algebra
Regularization
Smoothing Newton algorithm
Convergence analysis

ABSTRACT

The symmetric cone complementarity problem (denoted by SCCP) is a class of equilibrium optimization problems, and it contains the standard linear/nonlinear complementarity problem (LCP/NCP), the second-order cone complementarity problem (SOCCP) and the semidefinite complementarity problem (SDCP) as special cases. In this paper, we present a regularized smoothing Newton algorithm for SCCP by making use of Euclidean Jordan algebraic technique. Under suitable conditions, we obtain global convergence and local quadratic convergence of the proposed algorithm. Some numerical results are reported in this paper, which confirm the good theoretical properties of the proposed algorithm.

1. Introduction

In this paper, we consider the following symmetric cone complementarity problems (denoted by SCCP), which is to find a vector $x \in V$ such that

$$x \in \mathcal{K},\ y = F(x) \in \mathcal{K}, \quad \langle x, y \rangle = 0, \tag{1.1}$$

where V is an n-dimensional Euclidean space, $\mathcal{K} \subset V$ is a symmetric cone, $\langle \cdot, \cdot \rangle$ denotes the Euclidean inner product, and $F : V \to V$ is a continuously differentiable mapping.

SCCP (1.1) has wide applications. It is closely related to the optimality conditions for symmetric conic linear programming (SCLP) (see [1]). Furthermore, SCCP (1.1) includes semidefinite complementarity problems (SDCP), second-order cone complementarity problems (SOCCP), and linear/nonlinear complementarity problems (LCP/NCP) as special cases.

The research efforts have focused on interior methods for SCCP or more general for SCLP; see [1] and references therein. While interior-point methods appear successful in solving SCLP and SCCP, it is worth to explore other solution approaches for SCLP and SCCP. In particular, there has been recently active research on reformulation of smoothing Newton-type methods; see [2–8] for LCP/NCP, SOCCP and SDCP. Motivated by these developments, we study in this paper regularized smoothing Newton method for SCCP (1.1) based on the non-symmetrically perturbed smoothing Fischer–Burmeister function. Analogous to the analysis in semidefinite cone [6] and second-order cone [7,8], the definition of smoothing functions is based on the spectral factorization of Euclidean space V by the Jordan algebra associated with symmetric cone. For the analysis of the convergence properties of regularized smoothing Newton methods, we derive computable formulas for the non-symmetrically perturbed smoothing Fischer–Burmeister function and its Jacobian to study the Lipschitzian and differential properties of this function.

☆ This project is supported by National Natural Science Foundation of China (Grant No. 11071041) and Fujian Natural Science Foundation (Grant No. 2009J01002).
* Tel.: +86 13763827962.
E-mail address: macf@fjnu.edu.cn.

0895-7177/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.mcm.2011.06.013