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Application of the Hamiltonian approach to nonlinear vibrating equations

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ABSTRACT

In this paper, the Hamiltonian approach is applied to nonlinear vibrations and oscillations. Periodic solutions are analytically verified and consequently the relationship between the natural frequency and the initial amplitude is obtained in an analytical form. The method is applied to four nonlinear differential equations. It has indicated that by utilizing the Hamiltonian approach the first iteration leads us to a high accuracy of solutions.

The results obtained employing the Hamiltonian approach are compared with those achieved by using another analytical technique, named the Energy Balance Method (EBM) and also an accurate numerical solution to verify the accuracy of the proposed method. The results reveal that the Hamiltonian approach is very effective and simple. It is predicted that the Hamiltonian approach can prove versatile when confronted with engineering problems, as indicated in following examples. The obtained results may be useful for the explanation of some practical physical problems.

1. Introduction

Most phenomena in our world, which arise in engineering, are essentially nonlinear and are described by nonlinear equations. All these problems and phenomena are modeled by ordinary or partial differential equations. Except for a limited number of these problems, most of them do not have any analytical solution.

Nonlinear oscillators play a pivotal role in physics and engineering and have a significant importance in mechanical and structural dynamics for the comprehensive understanding and accurate prediction of motion.

There have been many analytical and numerical methods to solve the problems of nonlinear oscillators, such as the variational iteration method [1–5], energy balance method [6–15], coupled homotopy-variational formulation [16], variational approach [17–20] and the homotopy perturbation method [21–28].

Previously, He [12] introduced the energy balance method based on collocation and the Hamiltonian. This approach is very simple but strongly depends on the chosen location point. Recently, He [29] proposed the Hamiltonian approach. This approach is a kind of energy method with a vast application in conservative oscillatory systems.

In this paper, the Hamiltonian approach is applied to obtain the analytical approximate solution of the nonlinear vibrating equations and consequently the relationship between the natural frequency and the initial amplitude is obtained in an analytical form.

In order to clarify this approach, consider the following general oscillator [12,29]:

$$u'' + f(u, u', u'') = 0. \quad (1)$$

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