Analysis of second-order shear-deformable beams with semi-rigid connections

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ABSTRACT

In this paper a new beam finite element is proposed for the solution of framed structures with semi-rigid connections. The element uses the second order Timoshenko's beam model. Power series expansion of kinematic relations is employed to improve solution stability. Effect of flexible and eccentric connections is considered by means of rotational and linear springs plus rigid end-offset. The proposed approach has the same computational cost of a *standard* beam element since additional degrees of freedom are condensed out. The resulting element stiffness matrix expression is given as the summation of the second order Timohenko's beam stiffness matrix and corrective matrices. The proposed element is readily implementable in existing finite element codes. Some benchmark problems are used to test the efficiency of the proposed procedure.

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1. Introduction

Steel framework structural behavior is highly variable depending by the nature of the connection joining beam to columns. Common practice and wide experimental investigations have clearly demonstrated that most of these connections are semi-rigid. Therefore the adequate corresponding idealization has to be provided by a proper semi-rigid connections describers. Several are the models developed to account the effect of such type of connections. Briefly we can collect them in *empirical/experimental* models and *analytical/numerical* models [1]. Empirical models develop mathematical representations of the frame connections by means of the geometrical and mechanical properties. These methods require a fine calibration based on experimental tests. Analytical methods develop mathematical representations of the frame connections by idealizing the connection flexibility as ruled by a finite numbers of parameters. Given the above scenario here we present a new analytical method along the line presented in [2] and then further developed in [3–6] where a computer implementable second order classical beam finite element is presented and then further enriched to account for flexible and eccentric connections. Rotational springs are used to model the semi-rigid connections and rigid end-offset are used to model the joint dimension [7–12].

Several authors have proposed models for frame structures with semi-rigid connections [3–6] lately extended addressing to inelasticity property of steel material [13].

In this paper we focus the attention on the improvement of the theory of basic beam models: a new finite element model is presented for the analysis of steel frameworks. A Timoshenko's beam, whose stiffness matrix is computed by a power expression of trigonometric functions, is provided by a rotational spring and is considered to be ended by two rigid connections. Given the above mentioned attributes the model is supposed to be suitable for the study of short beams or composite beams where the shear deformations are consistent.

In this preliminary study, a linear relationship between moment and joint rotation is assumed. Next we will consider the effect, on the proposed model, of non-linear moment-rotation relationship, that is of paramount importance dealing with seismic design. Primary purpose of the present paper is to produce a finite element model readily implementable in extant commercial codes. To easily fulfill this requirement additional degrees of freedom are condensed-out in order to obtain a beam element with the same degrees of freedom used for a rigid connection [14,4]. This approach is known to be correct if the size of the connection is limited.

1.1. Overview

The paper is organized as follows. In the first section the governing equations of the problem are illustrated. In Section 2 we develop the formulation of the structural element by investigating the effect of eccentric and flexible connections. Corrective matrices for the semi-rigid approach are developed in order to obtain the final stiffness matrix as summation of each contribution. In this section the vector of generalized forces and the mass matrix are also presented. In Section 3 some numerical examples are given to test the accuracy of the presented model in the linear dynamic range. Some conclusive remarks end the paper.

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