Mathematical and Computer Modelling 54 (2011) 2418–2422

Contents lists available at ScienceDirect

Mathematical and Computer Modelling

journal homepage: www.elsevier.com/locate/mcm

A note on tvs-cone metric fixed point theory

Stojan Radenović [a,*], Suzana Simić [b], Nenad Cakić [c], Zorana Golubović [a]

[a] University of Belgrade, Faculty of Mechanical Engineering, Kraljice Marije 16, 11 120 Beograd, Serbia
[b] University of Kragujevac, Faculty of Science, Radoja Domanovića 12, 34000 Kragujevac, Serbia
[c] University of Belgrade, Faculty of Electrical Engineering, Bulevar Kralja Aleksandra 73, 11 120 Beograd, Serbia

ARTICLE INFO

Article history:
Received 28 October 2010
Received in revised form 24 May 2011
Accepted 25 May 2011

Keywords:
Tvs-cone metric
Tvs-cone metric space
Fixed point
Nadler's fixed point theorem
Locally convex space

ABSTRACT

In this paper, the concept of set-valued contraction of Nadler type in the setting of tvs-cone spaces was introduced and a fixed point theorem in the setting of tvs-cone spaces with respect to a solid cone was proved. Obtained results extend and generalize the main results of [S.B. Nadler Jr., Multi-valued contraction mappings, Pacific Journal of Mathematics 30 (1969) 475–488] and [D. Wardowski, On set-valued contractions of Nadler type in cone metric spaces, Applied Mathematics Letters 24 (2011) 275–278]. Two examples are given to illustrate the usability of the results.

1. Introduction and preliminaries

Huang and Zhang introduced in [1] the concept of cone metric spaces as a generalization of metric spaces. They have replaced the real numbers (as the co-domain of a metric) by an ordered Banach space. They described there the convergence in cone metric spaces, introduced their completeness and proved some fixed point theorems for contractive mappings on cone metric spaces. Recently in [2–19], many authors proved fixed point theorems in cone metric spaces.

Du [8] introduced the concept of *tvs*-cone metric and *tvs*-cone metric space to improve and extend the concept of cone metric space in the sense of Huang and Zhang [1]. In the papers [5,7,8,13], the authors tried to generalize this approach by using cones in topological vector spaces (*tvs*) instead of Banach spaces. However, it should be noted that an old result shows that if the underlying cone of an ordered *tvs* is solid and normal, then such *tvs* must be an ordered normed space. Thus, proper generalizations when passing from norm-valued cone metric spaces to *tvs*-valued cone metric spaces can be obtained only in the case of nonnormal cones (for details, see [13]).

We repeat some definitions and results from [13,14], which will be needed in the sequel.

Let E be a topological vector space with its zero vector θ. A nonempty subset P of E is called a convex cone if $P + P \subseteq P$ and $\lambda P \subseteq P$ for $\lambda \geq 0$. A convex cone P is said to be pointed (or proper) if $P \cap (-P) = \{\theta\}$; P is normal (or saturated) if E has a base of neighborhoods of zero consisting of order-convex subsets. For a given cone $P \subseteq E$, we can define a partial ordering $\preccurlyeq$ with respect to P by $x \preccurlyeq y$ if and only if $y - x \in P$; $x \prec y$ will stand for $x \preccurlyeq y$ and int$P$, where int$P$ denotes the interior of P. The cone P is said to be solid if it has a nonempty interior.

In the sequel, E will be a locally convex Hausdorff *tvs* with its zero vector θ, P a proper, closed and convex pointed cone in E with int$P \neq \emptyset$ and $\preccurlyeq$ a partial ordering with respect to P.

* Corresponding author.
E-mail addresses: sradenovic@mas.bg.ac.rs, radens@beotel.rs (S. Radenović), suzanasimic@kg.ac.rs (S. Simić), cakic@etf.rs (N. Cakić), zorana.golubovic@gmail.com (Z. Golubović).

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doi:10.1016/j.mcm.2011.05.051