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Controllability of pairs of matrices with prescribed entries[☆]

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ABSTRACT

Let F be an infinite field and let

$$(A_1, A_2) = \left(\begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix}, \begin{bmatrix} a_{1,3} \\ a_{2,3} \end{bmatrix} \right),$$

where the entries $a_{i,j} \in F, i \in \{1, 2\}, j \in \{1, 2, 3\}$. In this paper we establish necessary and sufficient conditions under which it is possible to prescribe some entries of $\begin{bmatrix} A_1 & A_2 \end{bmatrix}$, so that the pair (A_1, A_2) is completely controllable.

1. Introduction

Throughout the last decades, several results have been published describing the possibility of a pair of matrices being completely controllable, when some of its entries are known and the others are free, see for example [1–8]. In particular, in [9–11] we analyse the possibility of a pair of matrices of the form (A_1, A_2), where A_1 is square and $[A_1\ A_2]$ is partitioned into $(k-1) \times k$ blocks of size $p \times p$, to be completely controllable, when some of its blocks are fixed and the others vary. Indeed, we show that it is always possible to prescribe $k-1$ blocks of (A_1, A_2) and the property of (A_1, A_2) being completely controllable, except if, either the nonprincipal blocks of one row of $[A_1\ A_2]$ are prescribed equal to 0, or all the blocks of A_2 are prescribed equal to 0.

Our main purpose is to identify the maximum number of blocks that is possible to prescribe simultaneously with the property of (A_1, A_2) being completely controllable. This problem is very difficult and is still open. In order to give some insight into this question we start by studying the particular case where A_1 is of type 2×2 and A_2 is of type 2×1. Indeed, in the present paper we describe necessary and sufficient conditions under which the pair (A_1, A_2) is completely controllable, where

$$A_1 = \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix}, \qquad A_2 = \begin{bmatrix} a_{1,3} \\ a_{2,3} \end{bmatrix},$$

and the elements $a_{i,j}$ $(i \in \{1, 2\}, j \in \{1, 2, 3\})$ belong to an infinite field.

2. Background

Before we present our solution, we start by introducing some notation and results that are necessary for the rest of the paper. In general these results can be found in many books on Linear Algebra, see for example [12–17].

Let F be a field.

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