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Strong convergence theorems for nonlinear mappings, variational inequality problems and system of generalized mixed equilibrium problems

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ABSTRACT

In this paper, we construct a new iterative scheme by hybrid method and prove strong convergence theorems for approximation of a common element of set of common fixed points of an infinite family of relatively quasi-nonexpansive mappings, set of solutions to a variational inequality problem and set of common solutions to a system of generalized mixed equilibrium problems in a 2-uniformly convex real Banach space which is also uniformly smooth. Furthermore, using our iterative scheme, we prove strong convergence theorem to a common element of the set of fixed point of a weak relatively nonexpansive mapping, set of solutions to a variational inequality problem and the set of common solutions to a system of generalized mixed equilibrium problems. Finally, we give applications of our results in a Banach space. Our results extend many important recent results in the literature.

1. Introduction

Let E be a real Banach space with dual E^* and C be a nonempty, closed and convex subset of E. A mapping $T : C \to C$ is called *nonexpansive* if

$$\|Tx - Ty\| \leq \|x - y\|, \quad \forall x, y \in C. \tag{1.1}$$

A point $x \in C$ is called *a fixed point* of T if $Tx = x$. The set of fixed points of T is defined as $F(T) := \{x \in C : Tx = x\}$. A mapping $T : C \to C$ is called *quasi-nonexpansive* if

$$\|Tx - x^*\| \leq \|x - x^*\|, \quad \forall x \in C, \ x^* \in F(T).$$

It is clear that every nonexpansive mapping with nonempty set of fixed points is quasi-nonexpansive. We denote by J the normalized duality mapping from E to 2^{E^*} defined by

$$J(x) = \{f \in E^* : \langle x, f\rangle = \|x\|^2 = \|f\|^2\}.$$

The following properties of J are well known (The reader can consult [1–3] for more details):

(1) If E is uniformly smooth, then J is norm-to-norm uniformly continuous on each bounded subset of E.
(2) $J(x) \neq \emptyset, \ x \in E$.
(3) If E is reflexive, then J is a mapping from E onto E^*.
(4) If E is smooth, then J is single-valued.

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