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# Highly nonlinear neutral stochastic differential equations with time-dependent delay and the Euler–Maruyama method

Marija Milošević

*University of Niš, Faculty of Science and Mathematics, Višegradska 33, 18000 Niš, Serbia*

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ABSTRACT

The subject of this paper is the development of discrete-time approximations for solutions of a class of highly nonlinear neutral stochastic differential equations with time-dependent delay. The main contribution is to establish the convergence in probability of the Euler–Maruyama approximate solution without the linear growth condition, that is, under Khasminskii-type conditions. The presence of the delayed argument in the equation, especially in the derivative of the state variable, requires a special treatment and some additional conditions, except the conditions that guarantee the existence and uniqueness of the exact solution. The existence and uniqueness result and the convergence in probability are directly influenced by the properties of the delay function.

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## 1. Introduction and preliminary results

It is well known that the evolution of a physical system depending only on its present state and some random input can often be described by a stochastic ordinary differential equation. However, in many physical situations, the rate of change of the system depends not only on the present but also on its past states. In such cases, stochastic differential delay equations or stochastic functional differential equations provide an important tool for describing and analyzing such systems. A more general class of stochastic differential equations in which the delay argument occurs in the derivative of the state variable is the class of neutral stochastic differential equations. According to this fact, many theorems referring to stochastic delay and functional differential equations are successfully extended to neutral stochastic differential equations (see, for example, [1–5]). The fact that stochastic differential equations in most cases cannot be solved explicitly has been the main motivation for the development of different approximate methods such as those from the papers [6–12]. Moreover, in [13], the mean square convergence of the Euler–Maruyama numerical method for neutral stochastic functional differential equations is considered, under the local Lipschitz condition and the linear growth condition. As is known, stochastic differential equations with time-dependent delay can be regarded as a particular case of stochastic functional differential equations. On the other hand, they often have been treated separately, when the results are directly influenced by the properties of the delay function (see, for example [10,14]).

Many real-life phenomena are described by stochastic differential equations with coefficients that do not satisfy the linear growth condition. Thus, in papers [15–17], existence, uniqueness and stability of solutions to highly nonlinear stochastic delay differential equations and neutral stochastic differential equations are investigated. Moreover, in [18], Mao studied convergence in probability of the Euler–Maruyama solution for stochastic differential equations with constant delay under the generalized Khasminskii-type conditions.

The main aim of the present paper is to prove the convergence in probability of the Euler–Maruyama approximate solution for a class of highly nonlinear neutral stochastic differential equations with time-dependent delay. The paper is organized as follows.

*E-mail address:* 27marija.milosevic@gmail.com.

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