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On finite products of convolutions and classifications of hyperbolic and elliptic equations

Adem Kılıçman [a,*], Hassan Eltayeb [b]

[a] Department of Mathematics and Institute for Mathematical Research, University Putra Malaysia, 43400 UPM, Serdang, Selangor, Malaysia

[b] Mathematics Department, College of Science, King Saud University, P.O. Box 2455, Riyadh 11451, Saudi Arabia

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ABSTRACT

In this paper we consider linear second order partial differential equations with constant coefficients; then by using the single and double convolution products we produce some new equations with variable coefficients and we classify the new equations. It is shown that the classifications of the new equations are similar to the original equations that is, if the original equation is a hyperbolic then the new classification after convolution product is also a hyperbolic similarly, if an elliptic then the new classification is an elliptic equation. Thus we prove that the classifications of new equations are invariant after finite convolution products.

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1. Introduction

The topic of partial differential equations (PDEs) is probably the most important subject in applied mathematics since most of the mathematical modeling of a real engineering or physical system can be explained by using PDEs. From the pure mathematics perspective also the solution of the PDE(s) brings a very advanced level of knowledge such as eigenvalue problems, orthogonality, Laplace and Fourier Transform, linear algebra, several numerical methods and many more. In addition, there are a very rich variety of interesting applications ranging from engineering to finance.

At the same time, there is no general method to solve all the PDEs and the behavior of the solutions very much depends on the classification of PDEs therefore the problem of classification for PDEs is very natural and well known thus the classification governs the number and type of conditions to be prescribed in order that the problem is well posed and has a unique solution. On the other hand, many physical processes fundamental to science and engineering are governed by PDEs, for example Maxwell equations comprise a set of partial differential equations that form the basis of electromagnetic theory and are fundamental to physics, similarly, quantum mechanics is yet another theory governed by a PDE, the Schrödinger equation, which also forms the basis of much of physics, chemistry and electronic engineering, see [1].

However, the first of the major difficulties with PDEs is that it is extremely difficult to illustrate their solutions geometrically. A second basic problem with PDEs is that it is intrinsically more difficult to solve them or even to decide whether a solution exists.

There are three basic types of equations that appear in most areas of science and engineering and it is essential to understand their solutions before any progress can be made toward more complicated sets of equations such as nonlinear equations, or probably equations with variable coefficients.

* Corresponding author.
E-mail addresses: akilicman@putra.upm.edu.my (A. Kılıçman), hgadain@ksu.edu.sa (H. Eltayeb).

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