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On the generalization of some integral inequalities and their applications

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ABSTRACT

In this paper, a general integral identity for convex functions is derived. Then, we establish some new inequalities of the Simpson and the Hermite–Hadamard type for functions whose absolute values of derivatives are convex. Some applications for special means of real numbers are also provided.

1. Introduction

Let $f : I \subseteq \mathbb{R} \to \mathbb{R}$ be a convex mapping defined on the interval I of real numbers and $a, b \in I$ with $a < b$. The following double inequality:

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)\mathrm{d}x \leq \frac{f(a)+f(b)}{2} \tag{1.1}$$

is known in the literature as the Hadamard inequality for convex mapping. Note that some of the classical inequalities for means can be derived from (1.1) for appropriate particular selections of the mapping f. Both inequalities hold in the reversed direction if f is concave [1].

It is well known that the Hermite–Hadamard inequality plays an important role in nonlinear analysis. Over the last decade, this classical inequality has been improved and generalized in a number of ways; there have been a large number of research papers written on this subject, (see, [2–13]) and the references therein.

In [12], Sarikaya et al. established inequalities for twice differentiable convex mappings which are connected with Hadamard's inequality, and they used the following lemma to prove their results:

Lemma 1. *Let $f : I^\circ \subset \mathbb{R} \to \mathbb{R}$ be a twice differentiable function on I°, $a, b \in I^\circ$ (I° is the interior of I) with $a < b$. If $f'' \in L_1[a, b]$, then*

$$\frac{1}{b-a}\int_a^b f(x)\mathrm{d}x - f\left(\frac{a+b}{2}\right) = \frac{(b-a)^2}{2}\int_0^1 m(t)[f''(ta+(1-t)b) + f''(tb+(1-t)a)]\mathrm{d}t,$$

where

$$m(t) := \begin{cases} t^2, & t \in \left[0, \frac{1}{2}\right) \\ (1-t)^2, & t \in \left[\frac{1}{2}, 1\right]. \end{cases}$$

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