Mathematical and Computer Modelling 54 (2011) 2132–2138

Contents lists available at ScienceDirect

Mathematical and Computer Modelling

journal homepage: www.elsevier.com/locate/mcm

Solving a fractional order model of HIV infection of CD4$^+$ T cells

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ARTICLE INFO

Article history:
Received 27 February 2011
Received in revised form 12 May 2011
Accepted 12 May 2011

Keywords:
A model for HIV infection of CD4$^+$ T cells
Multi-step differential transform method
Fractional order ordinary differential equation systems

ABSTRACT

In this paper, a multi-step differential transform method (MsDTM) is performed to give approximate and analytical solutions of nonlinear fractional order ordinary differential equation systems such as a model for HIV infection of CD4$^+$ T cells. The numerical solutions obtained from the proposed method indicate that the approach is easy to implement and accurate when applied to systems of fractional differential equations. Some figures are presented to show the reliability and simplicity of the methods.

1. Introduction

In this paper, dynamics of a model for HIV infection of CD4$^+$ T cells is examined [1]. The components of the basic three component models are the concentration of susceptible CD4$^+$ T cells. CD4$^+$ T cells infected by the HIV viruses and free HIV virus particles in the blood are denoted respectively by $T(t)$, $I(t)$ and $V(t)$. CD4$^+$ T cells are also named as leukocytes or T helper cells. These with order cells in human immunity systems fight against diseases. HIV uses cells in order to propagate. The number of CD4$^+$ T cells of a healthy person is $\frac{800}{1200}$ mm^3. Here, a fractional order model of for HIV infection of CD4$^+$ T cells is reviewed. These quantities satisfy

$$\begin{aligned} D^{\alpha_1}T &= q - \alpha T + rT\left(1 - \frac{T+I}{T_{max}}\right) - k^*VT \\ D^{\alpha_2}I &= k^*VT - \beta I \\ D^{\alpha_3}V &= N\beta I - \gamma V \end{aligned} \tag{1}$$

with the initial conditions: $T(0) = r_1$, $I(0) = r_2$ and $V(0) = r_3$. Throughout this paper, we set $q = 0.1$, $\alpha = 0.02$, $\beta = 0.3$, $r = 3$, $\gamma = 2.4$, $k^* = 0.0027$, $N = 10$, $T_{max} = 1500$. The logistic growth of the healthy CD4$^+$ T cells is now described by $(1 - \frac{T+I}{T_{max}})$, and proliferation of infected CD4$^+$ T cells is neglected. The term kVT describes the incidence of HIV infection of healthy CD4$^+$ T cells, where $k > 0$ is the infection rate. Each infected CD4$^+$ T cell is assumed to produce N virus particles during its lifetime, including any of its daughter cells. The body is believed to produce CD4$^+$ T cells from precursors in the bone marrow and thymus at a constant rate q. When stimulated by antigen or mitogen, T cells multiply through mitosis with a rate r. T_{max} is the maximum level of CD4$^+$ T cell concentration in the body [2–5].

Differential equations of fractional order have been the subject of many studies owing to their implementation in various applications in fluid mechanics, viscoelasticity, biology, physics and engineering. Lately, a large amount of studies developed concerning the application of fractional differential equations in nonlinear dynamics [6–11]. As most fractional differential

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doi:10.1016/j.mcm.2011.05.022