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A finite iterative algorithm for solving the generalized (P, Q)-reflexive solution of the linear systems of matrix equations☆

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ABSTRACT

In this paper, we proposed an algorithm for solving the linear systems of matrix equations $\begin{cases} \sum_{i=1}^{N} A_i^{(1)} X_i B_i^{(1)} = C^{(1)}, \\ \vdots \\ \sum_{i=1}^{N} A_i^{(M)} X_i B_i^{(M)} = C^{(M)}. \end{cases}$ over the generalized (P, Q)-reflexive matrix $X_l \in R^{n\times m}$ ($A_l^{(i)} \in R^{p\times n}, B_l^{(i)} \in R^{m\times q}, C^{(i)} \in R^{p\times q}, l = 1, 2, \ldots, N, i = 1, 2, \ldots, M$). According to the algorithm, the solvability of the problem can be determined automatically. When the problem is consistent over the generalized (P, Q)-reflexive matrix X_l ($l = 1, \ldots, N$), for any generalized (P, Q)-reflexive initial iterative matrices $X_l(0)$ ($l = 1, \ldots, N$), the generalized (P, Q)-reflexive solution can be obtained within finite iterative steps in the absence of roundoff errors. The unique least-norm generalized (P, Q)-reflexive solution can also be derived when the appropriate initial iterative matrices are chosen. A sufficient and necessary condition for which the linear systems of matrix equations is inconsistent is given. Furthermore, the optimal approximate solution for a group of given matrices Y_l ($l = 1, \ldots, N$) can be derived by finding the least-norm generalized (P, Q)-reflexive solution of a new corresponding linear system of matrix equations. Finally, we present a numerical example to verify the theoretical results of this paper.

1. Introduction

Throughout the paper, R^n will denote the complex n-vector space and the set of $n \times m$ matrices by $R^{n\times m}$. For a matrix $A \in R^{m\times n}$, $\|A\|$ represents its Frobenius norm, $R(A)$ represents its column space, $tr(A)$ represents its trace and $vec(\cdot)$ represents the vec operator, i.e., $vec(A) = (a_1^T, a_2^T, \ldots, a_n^T)^T$ for the matrix $A = (a_1, a_2, \ldots, a_n) \in R^{m\times n}$, $a_i \in R^m$, $i = 1, 2, \ldots, n$. $A \otimes B$ stands for the Kronecker product of matrices A and B. In [1], the definition and some properties of generalized reflexive (anti-reflexive) matrix have been presented.

In [2], Peng and Hu presented the conditions for the solvability of matrix equation $AX = B$ over reflexive or anti-reflexive matrices and the conditions for the solvability of matrix equation $AXB = C$ over reflexive matrices have been presented in [3]. The sufficient and necessary conditions for the solvability of matrix equation $A^H XB = C$ over reflexive or anti-reflexive matrices were provided in [4]. By using the generalized singular value decomposition, a necessary and sufficient condition for the matrix equation $AXB = D$ over generalized reflexive matrices was given and the solution set was constructed explicitly when it is nonempty in [5]. In [6], the authors considered the generalized reflexive solutions for a class of matrix equations

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