Mathematical and Computer Modelling 54 (2011) 2109–2116

Contents lists available at ScienceDirect

Mathematical and Computer Modelling

journal homepage: www.elsevier.com/locate/mcm

Application of the $(\frac{G'}{G})$-expansion method for the Zhiber–Shabat equation and other related equations

A. Borhanifar *, Ali Zamiri Moghanlu

Department of Mathematics, University of Mohaghegh Ardabili, Ardabil, Iran

ARTICLE INFO

Article history:
Received 3 December 2009
Accepted 10 May 2011

Keywords:
$(\frac{G'}{G})$-expansion method
Zhiber–Shabat equation
Liouville equation
Dodd–Bullough–Mikhailov (DBM) equation
Tzitzeica–Dodd–Bullough (TDB) equation
Traveling wave solutions

ABSTRACT

In this paper, the exact traveling wave solutions of the Zhiber–Shabat equation and the related equations: Liouville equation, Dodd–Bullough–Mikhailov (DBM) equation, sinh–Gordon equation, and Tzitzeica–Dodd–Bullough (TDB) equation are studied by using the $(\frac{G'}{G})$-expansion method. Solitons and periodic solutions for these equations are formally derived.

1. Introduction

The study of exact solutions of nonlinear evolution equations plays an important role in the study of nonlinear physical phenomena. To obtain the traveling wave solutions, many methods were attempted, such as the differential transformation method [1], the sine–cosine method [2–6], the standard tanh and extended tanh methods [2–4,7–9], the exp-function method [10–15], the Jacobi elliptic function method [16], the inverse scattering method [17], Hirota's bilinear transformation [18], the tanh–sech method [19], homogeneous balance method [20–22], the Darboux transformation [23], and so on. One of the most effective and direct methods for constructing soliton solutions of nonlinear equations is the $(\frac{G'}{G})$-expansion method. The $(\frac{G'}{G})$-expansion method, first introduced by Wang et al. [24], has been widely used to search for various exact solutions of NLEEs [25–27]. The $(\frac{G'}{G})$-expansion method is based on the explicit linearization of nonlinear differential equations for traveling waves with a certain substitution which leads to a second-order differential equation with constant coefficients. The computations are performed with a computer algebra system to deduce few solutions of the nonlinear equations in an explicit form [28]. Recently, Wazwaz [29] investigated the nonlinear Zhiber–Shabat equation:

$$u_{xt} + pe^{u} + qe^{-u} + re^{-2u} = 0, \tag{1}$$

where p, q, and r, are arbitrary constants. For $q = r = 0$, Eq. (1) reduces to the Liouville equation. For $q = 0, r \neq 0$, Eq. (1) gives the well-known Dodd–Bullough–Mikhailov (DBM) equation. However, for $q \neq 0$, $r = 0$, Eq. (1) reduces to the sinh–Gordon equation. Moreover, for $p = 0, q = -1, r = -1$, we obtain the Tzitzeica–Dodd–Bullough (TDB) equation. These equations play a significant role in many scientific applications such as solid state physics, nonlinear optics, plasma physics, fluid dynamics, mathematical biology, nonlinear optics, dislocations in crystals, kink dynamics, and chemical kinetics, and quantum field theory [30–40]. Finding exact solutions for these nonlinear evolution equations, by using the $(\frac{G'}{G})$-expansion method, is our the goal.

* Corresponding author. Tel.: +98 451 5514703; fax: +98 451 5514701.
E-mail addresses: borhani@uma.ac.ir (A. Borhanifar), alizamiri63@gmail.com (A.Z. Moghanlu).

0895-7177/$ – see front matter
doi:10.1016/j.mcm.2011.05.020