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Adomian decomposition method for non-smooth initial value problems

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ABSTRACT

Adomian decomposition method is extended to the calculations of the non-differentiable functions. The iteration procedure is based on Jumarie's Taylor series. A specific fractional differential equation is used to elucidate the solution procedure and the results are compared with the exact solution of the corresponding ordinary differential equations, revealing high accuracy and efficiency.

1. Introduction

Many methods of mathematical physics have been developed to solve differential equations, among which the Adomian decomposition method is an efficient approximation technique to solve nonlinear models with initial-boundary value problems [1–3].

There are, however, few methods specifically developed for solving fractional differential equations (FDEs). All are adopted from methods for ordinary differential equations by suitable modifications, such as the variational iteration method [4], the Adomian decomposition method [3–7] and the homotopy perturbation method [8].

In all of the methods mentioned above, the solutions of the fractional differential equations should be analytical if the fractional derivative is in the Caputo sense. As is well known, fractal curves are everywhere continuous but nowhere differentiable. As a result, we cannot employ the Caputo derivative, which requires that the defined functions should be differentiable, to describe the motions in fractal time–space.

Recently, a modified Riemann–Liouville left derivative has been proposed by Jumarie [9,10]. Compared with the classical Caputo derivative, the definition of the fractional derivative is not required to satisfy higher integer-order derivative than α. Second, the αth derivative of a constant is zero. For these merits, Jumarie's modified derivative was successfully applied in the probability calculus [9], fractional Laplace problems [10], fractional variational calculus [11], fractional variational iteration method [12] and fractional Taylor series [13].

In this paper, we extend the famous Adomian decomposition method to deal with the fractional initial problems by implementing Jumarie's fractional Taylor series [13].

2. Properties of modified Riemann–Liouville derivative

Assume $f : R \to R, x \to f(x)$ denote a continuous (but not necessarily differentiable) function and let the partition $h > 0$ in the interval [0, 1]. Through the fractional Riemann–Liouville integral

$$ {}_0I_x^{\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_0^x (x-\xi)^{\alpha-1} f(\xi)\mathrm{d}\xi, \quad \alpha > 0, \tag{1} $$

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