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Generalisation of the Lagrange multipliers for variational iterations applied to systems of differential equations

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ABSTRACT

In this paper, a new approach to the variational iteration method is introduced to solve systems of first-order differential equations. Since higher-order differential equations can almost always be converted into a first-order system of equations, the proposed method is still applicable to a wide range of differential equations. This generalised approach, unlike the classical method, uses restricted variations only for nonlinear terms by generalising the Lagrange multipliers. Consequently, this allows us to use the well known, but ignored, theory of linear ODEs for computing the matrix-valued Lagrange multipliers.

In order to validate the newly proposed approach in solving linear and nonlinear systems of differential equations, illustrative examples are presented: it turns out that the use of the generalised Lagrange multipliers is more reliable and efficient.

1. Introduction

It is well known that the dynamical behaviour of real-life problems can be described by a system of first-order differential equations. However, it is in general difficult to solve such types of systems. Finding approximate, and if possible in closed-form, solutions of differential equations is the subject of many researchers.

The Variational Iteration Method (VIM) was developed by Chinese mathematician Ji-Huan He in 1997 [1–4]. VIM has been applied to the Klein–Gordon equation [5], the Helmholtz equation [6], differential algebraic equations [7], epidemic and prey–predator models [8], the hyperchaotic Rossler system [9], nonlinear boundary value problems [10], and many others [2–4,11–22].

Over the years, different modifications of VIM have been proposed. In [23], Batiha et al. proposed the Multistage Variational Iteration Method (MVIM); in [24], Odibat developed modifications of VIM to approximate the solutions of nonlinear problems; in [25], Turkyilmazoglu proposed an optimal variational iteration method; in [26], Salkuyeh applied the method to linear systems of ordinary differential equations with constant coefficients; another modification of VIM was proposed in [27]. For a recent brief review of the method as well as its modified algorithms we refer to [28].

In the present paper we propose a new approach to the variational iteration method to solve systems of first-order differential equations. The new approach obtains the solutions of these systems without the need of restricted variations in the linear terms. By generalising the classical (scalar-valued) Lagrange multipliers to matrix-valued functions, the new approach emphasises the correct usage of the method and reduces the computational size and the number of iterations in many applications of the technique.

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