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The l_1 exact G-penalty function method and G-invex mathematical programming problems

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ABSTRACT

In this paper, we consider G-invex mathematical programming problems and show the efficiency of G-invexity notion in proving optimality results for such nonconvex optimization problems. Further, we introduce a new exact penalty function method, called the l_1 exact G-penalty function method and use it to solve nonconvex mathematical programming problems with G-invex functions. In this method, the so-called exact G-penalized optimization problem associated with the original optimization problem is constructed. The equivalence between the sets of optimal solutions of the original mathematical programming problem and of its associated G-penalized optimization problem is established under suitable G-invexity assumptions. Also lower bounds on the penalty parameter are given, for which above this result is true. It turns out that, for some nonconvex optimization problems, it is not possible to prove the same result for the classical l_1 penalty function method under invexity assumption.

1. Introduction

In the paper, we consider the following constrained optimization problem:

$$\begin{aligned} &\text{minimize } f(x) \\ &\text{subject to} \quad g_i(x) \leq 0,\ i \in I = \{1, \ldots, m\}, \\ &h_j(x) = 0,\ j \in J = \{1, \ldots, s\}, \\ &x \in X, \end{aligned} \tag{P}$$

where $f : X \to R$ and $g_i : X \to R$, $i \in I$, $h_j : X \to R$, $j \in J$, are differentiable functions on a nonempty open set $X \subset R^n$.

We will write $g := (g_1, \ldots, g_m) : X \to R^m$ and $h := (h_1, \ldots, h_s) : X \to R^s$ for convenience.

For the purpose of simplifying our presentation, we will next introduce some notation which will be used frequently throughout this paper.

Let

$$D := \left\{x \in X : g_i(x) \leq 0,\ i \in I, h_j(x) = 0,\ j \in J\right\}$$

be the set of all feasible solutions of (P).

Further, we denote the set of active constraints at point $\overline{x} \in X$,

$$I(\overline{x}) = \{i \in I : g_i(\overline{x}) = 0\}.$$

Considerable attention has been given in the recent years to devising methods for solving nonlinear programming problems via unconstrained minimization techniques. One class of methods which has emerged as very promising is the class of exact penalty functions methods. These functions have quite different features. One important property that

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