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# Convergence theorems by hybrid method for systems of equilibrium problems and fixed point problem

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ABSTRACT

The purpose of this paper is to introduce a hybrid algorithm for finding a common element of the set of common fixed points of two relatively quasi-nonexpansive mappings and the set of solutions of a system of equilibrium problems in a uniformly smooth and strictly convex real Banach space which also has Kadec–Klee property using the properties of generalized $f$-projection operator. Our results extend important recent results.



## 1. Introduction

Let $E$ be a real Banach space and $C$ be nonempty, closed and convex subset of $E$. A mapping $T : C \to C$ is called *nonexpansive* if

$$\|Tx - Ty\| \leq \|x - y\|, \quad \forall x, y \in K. \tag{1.1}$$

A point $x \in C$ is called *a fixed point* of $T$ if $Tx = x$. The set of fixed points of $T$ is defined as $F(T) := \{x \in C : Tx = x\}$.

A mapping $T : C \to C$ is called *quasi-nonexpansive* if

$$\|Tx - x^*\| \leq \|x - x^*\|, \quad \forall x \in C,\ x^* \in F(T).$$

It is clear that every nonexpansive mapping with nonempty set of fixed points is quasi-nonexpansive.

In [1], Matsushita and Takahashi introduced a hybrid iterative scheme for approximation of fixed points of relatively nonexpansive mapping in a uniformly convex real Banach space which is also uniformly smooth: $x_0 \in C$,

$$\begin{cases} y_n = J^{-1}(\alpha_n Jx_n + (1-\alpha_n)JTx_n), \\ H_n = \{w \in C : \phi(w, y_n) \leq \phi(w, x_n)\}, \\ W_n = \{w \in C : \langle x_n - w, Jx_0 - Jx_n\rangle \geq 0\}, \\ x_{n+1} = \Pi_{H_n \cap W_n} x_0, \quad n \geq 0. \end{cases} \tag{1.2}$$

They proved that $\{x_n\}_{n=0}^{\infty}$ converges strongly to $\Pi_{F(T)}x_0$, where $F(T) \neq \emptyset$.

Recently, Qin et al. [2] proved a strong convergence theorem for finding a common element of the set of common fixed points of two relatively quasi-nonexpansive mappings and the set of solution of an equilibrium problem in a uniformly convex and uniformly smooth real Banach space. In particular, they proved the following theorem.

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