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Approximating electrical distribution networks via mixed-integer nonlinear programming

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ABSTRACT

Given urban data derived from a geographical information system (GIS), we consider the problem of constructing an estimate of the electrical distribution system of an urban area. We employ the image data to obtain an approximate electrical load distribution over a network of a prespecificed discretization. Together with partial information about existing substations, we determine the optimal placement of electrical substations to sustain such a load that minimizes the cost of capital and losses. This requires solving large-scale quadratic programs with discrete variables for which we present a novel penalization-smoothing scheme. The choice of locations allows one to determine the optimal flows in this network, as required by physical requirements which provide us with an approximation of the distribution network. Furthermore, the scheme allows for approximating systems in the presence of *no-go* areas, such as lakes and fields. We examine the performance of our algorithm on the solution of a set of location problems and observe that the scheme is capable of solving large-scale instances, well beyond the realm of existing mixed-integer nonlinear programming solvers. We conclude with a case study in which a stage-wise extension of this scheme is developed to reflect the temporal evolution of load.

1. Introduction

Obtaining information pertaining to a city's utility networks is crucial for purposes of planning, maintenance and redesign. Yet, this information is often inaccessible to most agencies and a relevant question is how one may approximate such information through the usage of only image data. This provides a motivation for estimating the underlying distribution system of an urban area, given an image from a geographical information system (GIS). Such an image captures the parcel data as well as a subset of existing substations in the urban area. We consider a novel approach that comprises of two basic steps. First, the input image is used to construct an electrical load distribution on a grid of prespecified discretization. Note that a grid, in this context, refers to a regular fully connected network over which a distribution network will be specified. Second, we solve an inverse problem that estimates the set of lines that correspond to such a load distribution. Several issues complicate such an estimation. In general, the true distribution system is a radial network and the resulting inverse problem falls within the realm of optimization problems in function space and is, in general, intractable. Instead, we restrict the set of possible networks to those that can be specified as graphs on a grid of chosen size. Two additional complexities emerge from modeling distribution systems in urban areas. First, there are significant areas that cannot be covered by the distribution system (such as lakes or fields). Therefore, the optimal solution has to reflect these restrictions. Second, a clear evolution pattern exists in the growth of the load and needs to be respected in estimating the distribution system. For instance, if a particular part of a township developed earlier than another, then the distribution system would have such a structure.

The resulting problem can be recast as an mathematical program in finite-dimensional space in which one seeks a set of flows that satisfy the substation capacity constraints, Kirchhoff's conservation equations and voltage bounds. Unfortunately, this problem can be infeasible if the substation information is inaccurate. To avert this possibility, we consider a problem in which we determine the installation of incremental substations as well as the flows that emerge from the resulting system. This optimization problem falls within a class of mixed-integer nonlinear problems (MINLP) and has a size that grows with the level of discretization and the number of substations. Currently no solvers exist for

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