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Subclasses of spirallike multivalent functions

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ABSTRACT

We studied convolution properties of spirallike, starlike and convex functions, and some special cases of the main results are also pointed out. Further, we also obtained inclusion and convolution properties for some new subclasses on p-valent functions defined by using the Dziok–Srivastava operator.

1. Introduction and preliminaries

Let $\mathcal{A}_p$ denote the class of functions of the form

$$f(z) = z^p + \sum_{k=p+1}^{\infty} a_k z^k, \quad (p \in \mathbb{N} = \{1, 2, \ldots\}), \tag{1.1}$$

which are analytic and p-valent in the open unit disc $\mathrm{U} = \{z \in \mathbb{C} : |z| < 1\}$.

If f and g are analytic functions in U, we say that f is *subordinate* to g, written $f(z) \prec g(z)$, if there exists a *Schwarz function* w, which (by definition) is analytic in U with $w(0) = 0$, and $|w(z)| < 1$ for all $z \in \mathrm{U}$, such that $f(z) = g(w(z))$, for all $z \in \mathrm{U}$. Furthermore, if the function g is univalent in U, then we have the following equivalence:

$$f(z) \prec g(z) \Leftrightarrow f(0) = g(0) \quad \text{and} \quad f(\mathrm{U}) \subset g(\mathrm{U}).$$

We will define the following two subclasses of a multivalent function:

Definition 1.1. Let $\lambda \in \mathbb{R}$ with $|\lambda| < \frac{\pi}{2}$, let $p \in \mathbb{N}$, and let ϕ be a univalent function in the unit disc U with $\phi(0) = 1$, such that

$$\operatorname{Re} \phi(z) > 1 - \frac{1}{p}, \quad z \in \mathrm{U}. \tag{1.2}$$

We define the classes $\mathcal{S}_p^\lambda(\phi)$ and $\mathcal{C}_p^\lambda(\phi)$ by

$$\mathcal{S}_p^\lambda(\phi) = \left\{ f \in \mathcal{A}_p : \mathrm{e}^{i\lambda} \frac{zf'(z)}{f(z)} \prec p\left(\phi(z)\cos\lambda + i\sin\lambda\right) \right\}$$

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