



## Structural stability and decay estimate for marine riser equations

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### ABSTRACT

We study the problem of the continuous dependence of weak and strong solutions to the initial boundary value problem for multidimensional marine riser equations on the parameters of the equation. The continuous dependence of solutions on the coefficient of the effective tension, on the coefficient of the nonlinear drag, and on the coefficients of the Coriolis force is established.

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### 1. Introduction

In recent years there has been much interest in the study of the question of continuous dependence of solutions to initial boundary value problems for various partial differential equations on coefficients in the equations. These kinds of problem are also called structural stability problems. We refer to the books [1,2], the papers [3–7], and references therein for relevant results.

We study the question of structural stability for the following initial boundary value problem for the multidimensional marine riser equation:

$$u_{tt} + k\Delta^2 u + a\Delta u + \vec{g} \cdot \nabla u_t + b|u_t|^p u_t = 0, \quad x \in \Omega, \quad t > 0, \quad (1.1)$$

$$u(x, 0) = u_0(x), \quad u_t(x, 0) = u_1(x), \quad x \in \Omega \quad (1.2)$$

$$u = \frac{\partial u}{\partial \nu} = 0, \quad x \in \partial\Omega, \quad t > 0, \quad (1.3)$$

where  $\Omega \subset \mathbb{R}^N$ ,  $N \leq 3$  is a bounded domain with sufficiently smooth boundary  $\partial\Omega$ ,  $\nu$  is the unit outward normal vector to the boundary,  $k > 0$ ,  $p \geq 1$ ,  $b > 0$ ,  $a \in \mathbb{R}$ , are given numbers, and  $\vec{g} = (g_1, \dots, g_N) \in \mathbb{R}^N$  is a given vector.

This problem is a multidimensional version of the following problem modeling the dynamics of marine risers:

$$mu_{tt} + Elu_{xxxx} - (N_{\text{eff}}u_x)_x + \gamma u_{tx} + bu_t|u_t| = 0, \quad x \in (0, l), \quad t > 0, \quad (1.4)$$

$$u(0, t) = u_{xx}(0, t) = u(l, t) = u_{xx}(l, t) = 0, \quad t > 0, \quad (1.5)$$

where  $El$  is the flexural rigidity of the riser,  $N_{\text{eff}}$  is the coefficient of the effective tension,  $\gamma$  is the Coriolis force,  $b$  is the nonlinear drag force, and  $m$  represents the mass line density. By using the Lyapunov function technique, it is shown in [8] that the zero solution of the problem is stable. In [9], the initial boundary value problem for the equation

$$mu_{tt} + ku_{xxxx} - [a(x)u_x]_x + \gamma u_{tx} + bu_t|u_t|^p = 0 \quad (1.6)$$

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