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On approximating weakly/properly efficient solutions in multi-objective programming

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ABSTRACT

This paper deals with approximate solutions of general (that is, without any convexity assumption) multi-objective optimization problems (MOPs). In this text, by reviewing some standard scalarization techniques we are interested in finding the relationships between ε-(weakly, properly) efficient points of an MOP and ϵ-optimal solutions of the related scalarized problem. For this purpose, the relationships between $\epsilon \in \mathbb{R}_{\geqq}$ and $\varepsilon \in \mathbb{R}^m_{\geqq}$, for a single objective and multi-objective problems, respectively, are analyzed. In fact, necessary and/or sufficient conditions for approximating (weakly, properly) efficient points of a general MOP via approximate solutions of the scalarized problems are obtained.

1. Introduction

In the recent years, the concept of approximate solution for an MOP has been studied in several frameworks and with kinds of purposes. The causes for interest in approximately efficient points of an MOP are the following actualities. First, iterative algorithms or heuristic algorithms will provide approximate solutions anyhow. Second, the efficient set may be empty, but under very weak requirements, approximate efficient points can be nonempty. The first and most fashionable notion of approximation was suggested by Kutateladze [1] (this concept will be used in this paper). Thereafter, the notion of ε-approximation from scalar problems to MOPs was expanded by Loridan [2]. White [3] proposed six kinds of ε-approximate notions for the MOPs. In the past two decades, a number of other authors have investigated ε-efficiency in MOPs; see for example [4–11] and references therein. Recently, Gutierrez et al. [12] have presented a new approximation concept and provided necessary and sufficient conditions for ε-(weakly) efficient points of an MOP. In fact, by defining a new approximation concept, they generalized the results obtained in [13] to an arbitrary ordered cone.

To a determinant, good approximate points are very applicable and beneficial in decision making; hence, obtaining ε-efficient points is very important. Nevertheless, similar to efficient points, there are also ε-efficient points with unbounded trade-off between criteria. Thus, it is necessary to filter out the ε-efficient points with unbounded trade-off between objectives and keep so called ε-properly efficient points.

Li and Wang [14] introduced the concept of ε-proper efficiency and, using specified scalarization techniques, established necessary conditions for ε-properly efficient points of a general MOP. Thereafter, Liu [15] attained a necessary and sufficient condition for ε-properly efficient points of a convex MOP. Recently, Beldiman et al. [16] have introduced $(\varepsilon, \overline{\varepsilon})$-properly efficient points, a generalization of definition given by Li and Wang [14], and obtained necessary conditions for these points. Gao et al. [17] presented a new definition for εq-proper efficiency and considered first order and second order necessary and sufficient conditions for εq-properly efficient points of a multi-objective problem. More recently, Gao et al. [18] have introduced a new definition for approximating properly efficient points of a vector optimization problem, which is not based on the Kutateladze [1] concept of approximate efficiency. Gao et al. [18], through scalarization, obtained necessary and sufficient conditions for their approximate properly efficient points.

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