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Efficient solution algorithms for computing fold points of power flow equations

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ABSTRACT

Techniques for identifying fold points of power flow equations are fundamental to voltage stability analysis. This paper presents efficient methods for computing fold points of power flow equations. The proposed methods make use of two generalized extended systems for determining fold points of nonlinear systems, i.e., the Moore–Spence system and the minimally extended system. Block elimination (BE) techniques are used in the solution procedures for the two systems. Hence, the sparsity of Jacobian matrix of power flow equations can be fully exploited to enhance computational efficiency. Left and right singular vectors corresponding to the zero eigenvalue of Jacobian matrix of nonlinear power flow equations can be obtained simultaneously at the end of the iterations. Numerical examples of Ward & Hale 6-bus system and a real 1620-bus power system in China are presented to validate the methods.

1. Introduction

Bifurcation theory studies the solution behavior of nonlinear parameter-dependent system [1]. It has been recognized as a fundamental tool to study voltage stability. Voltage collapse in power system has been explained by bifurcation problems of nonlinear power flow equations used to model the system, particularly fold points (or called limit points, turning points, saddle-node bifurcation points, etc.) which occur when a stationary point loses stability and coalesces with an unstable point as a parameter varies [2].

Techniques for calculating fold points of power flow equations play an important role in voltage stability analysis. A set of fold points define the boundary between the regions of stable and unstable operation, and these points on the boundary are sometimes referred to as maximum loading points (MLP) in power system. Accurate determination of fold points of power flow equations provides the information of loading margin to voltage collapse. It also forms the basis of other research aspects in voltage stability analysis, such as loading margin sensitivity analysis [3,4], multi-parameter bifurcation analysis [5,6], etc.

At or near fold points, the Jacobian matrix of power flow equations becomes singular, and consequently, the conventional Newton's method will show numerical difficulties. Numerical bifurcation techniques provide the methods to solve this problem, e.g., for calculating fold points of the power flow equations, continuation methods [7–9] and direct methods [10–12] are applied. In power system, some authors [13,14] also used optimization methods to tackle the problem.

Continuation methods or continuation power flow are widely known as powerful and useful methods of approximating fold points of nonlinear power flow equations. The methods use predictor–corrector scheme to trace the solution curves of power flow equations as the parameter varies. Fold points on the solution curves can be identified through monitoring certain test functions during continuation procedures. Continuation methods do not provide a tool to calculate fold points accurately since the basis of predictor–corrector scheme is an incremental procedure. If fold points have to be computed within a certain accuracy, a bisection method may slow down the continuation process [1].

Direct methods present an efficient way of accurately locating fold points of power flow equations in one step and may provide left or right singular vectors at the same time. Numerical methods for the straightforward calculation of fold points are based on so-called extended systems. For n-dimensional power flow equations, the $(2n+1)$-dimensional Moore–Spence system [15] is primarily used at present to determine fold points [10–12]. The approach using $(2n+1)$-dimensional system is also called point of collapse method in Refs. [7,10]. It would appear that the computational effort involved in solving the Moore–Spence system increases considerably when n becomes larger, so special techniques are required in the implementation to solve the Moore–Spence system.

A general numerical method using bordered matrices for the study of fold points and other high-order singularities of large nonlinear systems was presented by Govaerts [16]. This involves

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