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# Some $I$-convergent lambda-summable difference sequence spaces of fuzzy real numbers defined by a sequence of Orlicz functions

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ABSTRACT

In this paper we introduce certain new sequence spaces of fuzzy numbers defined by $I$-convergence using sequences of Orlicz functions and a difference operator of order $m$. We study some basic topological and algebraic properties of these spaces. Also we investigate the relations related to these spaces.



## 1. Introduction

The concept of fuzzy sets was initially introduced by Zadeh [1]. It has a wide range of applications in almost all branches of study, in particular in science, where mathematics is used. Now the notion of fuzziness is used by many researchers in cybernetics, artificial intelligence, expert systems and fuzzy control, pattern recognition, operations research, decision making, image analysis, projectiles, probability theory, agriculture, weather forecasting, etc. The fuzziness of all the subjects of mathematical sciences has been investigated. It has attracted many workers on sequence spaces and summability theory to introduce different types of fuzzy sequence spaces and to study their different properties. Our studies are based on the linear spaces of sequences of fuzzy numbers which are very important for higher level studies in quantum mechanics, particle physics and statistical mechanics, etc. Different classes of sequences of fuzzy real numbers have been discussed by Nanda [2], Nuray and Savas [3], Matloka [4], Altinok et al. [5], Colak et al. [6] and many others.

The notion of $I$-convergence was initially introduced by Kostyrko et al. [7]. Later on, it was further investigated from the sequence space point of view and linked with the summability theory by Šalàt et al. [8,9], Tripathy and Hazarika [10,11], Kumar and Kumar [12], Savas [13,14] and many other authors.

Let $X$ be a non-empty set, then a family of sets $I \subset 2^X$ (the class of all subsets of $X$) is called an *ideal* if and only if for each $A, B \in I$, we have $A \cup B \in I$ and for each $A \in I$ and each $B \subset A$, we have $B \in I$. A non-empty family of sets $F \subset 2^X$ is a *filter* on $X$ if and only if $\Phi \notin F$, for each $A, B \in F$, we have $A \cap B \in F$ and for each $A \in F$ and each $A \subset B$, we have $B \in F$. An ideal $I$ is called a *non-trivial* ideal if $I \neq \Phi$ and $X \notin I$. Clearly $I \subset 2^X$ is a non-trivial ideal if and only if $F = F(I) = \{X - A : A \in I\}$ is a filter on $X$. A non-trivial ideal $I \subset 2^X$ is called *admissible* if and only if $\{\{x\} : x \in X\} \subset I$. A non-trivial ideal $I$ is maximal if there cannot exist any non-trivial ideal $J \neq I$ containing $I$ as a subset. Further details on ideals of $2^X$ can be found in [7].

Let $w_F$ be the set of all sequences of fuzzy numbers. The operator $\Delta^n : w_F \to w_F$ is defined by $(\Delta^0 X)_k = X_k$; $(\Delta^1 X)_k = \Delta X_k = X_k - X_{k+1}$; $(\Delta^n X)_k = \Delta^n X_k = \Delta^{n-1} X_k - \Delta^{n-1} X_{k+1}$, $(n \geq 2)$ for all $n \in \mathbf{N}$. The generalized difference has the following

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