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Some iterative methods for general nonconvex variational inequalities

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ABSTRACT

In this paper, we suggest and analyze some three-step iterative methods for solving general nonconvex variational inequalities using the technique of updating the solution. We show that the convergence of these iterative methods requires only the partially relaxed strongly monotonicity which is a weaker condition than cocoerciveness. We are also discuss several special cases. Our method of proof is very simple compared with other techniques.

1. Introduction

General variational inequalities involving two operators were introduced and studied by Noor [1] in 1988. It turned out that a wide class of nonsymmetric and odd-order unrelated problems, which arise in various branches of pure and applied sciences can be studied in the unified framework of general variational inequalities. General variational inequalities can be considered as a significant and novel generalization of the variational inequalities, which were introduced and studied by Stampacchia [2] in 1964. For applications, physical formulation, numerical methods and other aspects of variational inequalities, see [3–9,1,10–30,2] and the references therein. However, all the work carried out in this direction assumed that the underlying set is a convex set. In many practical situations, a choice set may not be a convex set so that the existing results may not be applicable. To handle such situations, Noor [23] has introduced and considered a new class of variational inequalities, called the general nonconvex variational inequality on the uniformly prox-regular sets. It is well-known that uniformly prox-regular sets are nonconvex and include the convex sets as special cases, see [6,7,30]. Using the projection operator, Noor [18] has established the equivalence between the general nonconvex variational inequalities and the fixed point problem. In this paper, we prove a new characterization of the projection operator for the prox-regular sets. Using this characterization, one can easily show that the nonconvex projection operator is Lipschitz continuous, which is a new result. Using the equivalence between the nonconvex general variational inequalities, we suggest a unified extragradient method for solving the nonconvex general variational inequalities, which include the modified projection method of Noor [23] and the extragradient method of Korpelevich [9] as special cases. In this paper, we use the technique of updating the solution to suggest and analyze some three-step iterative methods for solving the general nonconvex variational inequalities. It has been shown that the convergence of three-step iterative methods for solving the general nonconvex variational inequalities requires that the operator must be strongly monotone and Lipschitz continuous. These several restrictions rule out its applications in various problems. The main motivation of this paper is to improve this criteria. We show that the convergence of three-step iterative method only requires only the partially relaxed strongly monotonicity, which is a weaker condition

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