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The local discontinuous Galerkin finite element method for Burger's equation[☆]

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ABSTRACT

In this paper, we study the local discontinuous Galerkin (LDG) finite element method for solving a nonlinear Burger's equation with Dirichlet boundary conditions. Based on the Hopf–Cole transformation, we transform the original problem into a linear heat equation with Neumann boundary conditions. The heat equation is then solved by the LDG finite element method with special chosen numerical flux. Theoretical analysis shows that this method is stable and the $(k+1)$th order of convergence rate when the polynomials $\mathcal{P}^k$ are used. Finally, we present some examples of $\mathcal{P}^k$ polynomials with $1 \leq k \leq 4$ to demonstrate the high-order accuracy of this method. The numerical results are also shown to be more accurate than some available results given in the literature.

1. Introduction

In this paper, we consider the following nonlinear Burger's equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \nu\frac{\partial^2 u}{\partial x^2}, \quad x \in (0, 1),\ t \in (0, T] \tag{1}$$

with initial condition

$$u(x, 0) = f(x), \quad 0 < x < 1 \tag{2}$$

and boundary conditions

$$u(0, t) = 0, \qquad u(1, t) = 0, \quad t \in (0, T] \tag{3}$$

where $\nu = \frac{1}{Re}$ and Re is the Reynolds number characterizing the size of viscosity. Burger's equation can be considered as a model equation for the decay of turbulence within a box of length L. Physical boundary conditions require u to be zero at the ends of the box, so that $u \to 0$ as $x \to 0$ and $x \to L$.

Burger's equation (1) is an important and basic partial differential equation from fluid mechanics, and has been widely used for various applications, such as modeling of gas dynamics and traffic flow, describing wave propagation in acoustics and hydrodynamics, etc. For a set of special initial and boundary conditions, Burger's equation can be analytically solved. In the context of gas dynamics, Hopf [1] and Cole [2] have independently shown that, for any initial condition, Burger's equation

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