The starlikeness of analytic functions of Koebe type

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ABSTRACT

We investigate the new class $\mathcal{H}_b(\alpha, \beta)$ of analytic functions with Koebe type. The subordination, superordination, best dominant result for that class was obtain. And by making use of Jack's Lemma as well as several differential and other inequalities, the authors derive sufficient conditions for starlikeness of the class $\mathcal{H}_b(\alpha, \beta)$ of n-fold symmetric analytic functions of Koebe type. Relevant connections of the results presented here with those given in earlier works are also indicated.

1. Introduction

Let denote by $H(\mathbb{U})$ the space of all analytical functions in the unit disk $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$. If $f, F \in H(\mathbb{U})$ and F is univalent in U we say that the function f is *subordinate* to F, or F is *superordinate* to f, written $f(z) \prec F(z)$, if $f(0) = F(0)$ and $f(\mathbb{U}) \subseteq F(\mathbb{U})$.

For $a \in \mathbb{C}$ and $n \in \mathbb{N}^*$ we denote

$$H[a, n] = \{f \in H(\mathbb{U}) : f(z) = a + a_n z^n + \cdots\}.$$

Letting $\varphi : \mathbb{C}^3 \times \overline{\mathbb{U}} \to \mathbb{C}, h \in H(\mathbb{U})$ and $q \in H[a, n]$, in [1] the authors determined conditions on φ such that

$$h(z) \prec \varphi(p(z), zp'(z), z^2 p''(z); z) \quad \text{implies} \quad q(z) \prec p(z),$$

for all p functions that satisfy the above superordination. Moreover, they found sufficient conditions so that the q function is the *largest* function with this property, called the *best subordinant* of this superordination.

Let $\mathcal{A}$ denote the class of normalized analytic functions of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k, \tag{1.1}$$

which are analytic in the open unit disk $\mathbb{U} = \{z : |z| < 1\}$. Also, as usual, let

$$S^* = \left\{ f : f \in \mathcal{A} \text{ and } \operatorname{Re}\left(\frac{zf'(z)}{f(z)}\right) > 0, \ (z \in \mathbb{U}) \right\}, \tag{1.2}$$

and

$$ST = \left\{ f : f \in \mathcal{A} \text{ and } \left|\arg\left(\frac{zf'(z)}{f(z)}\right)\right| < \frac{\alpha\pi}{2}, \ (z \in \mathbb{U}), (0 < \alpha \leq 1) \right\}, \tag{1.3}$$

be the familiar classes of *starlike functions* in $\mathbb{U}$ and *strongly starlike functions of order α* in $\mathbb{U}$, respectively.

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