Mathematical and Computer Modelling 54 (2011) 2923–2927

Contents lists available at SciVerse ScienceDirect

Mathematical and Computer Modelling

journal homepage: www.elsevier.com/locate/mcm

Fixed points for generalized weakly contractive mappings in partial metric spaces

Thabet Abdeljawad

Cankaya University, Department of Mathematics and Computer Science, 06530, Ankara, Turkey

ARTICLE INFO

Article history:
Received 14 February 2011
Received in revised form 11 July 2011
Accepted 11 July 2011

Keywords:
Partial metric space
Generalized weakly contractive
Fixed point
Common fixed point
Alternating distance function
Property (P)
Property (Q)

ABSTRACT

Partial metric spaces were introduced by S. G. Matthews in 1994 as a part of the study of denotational semantics of dataflow networks. In this article, we prove fixed point theorems for generalized weakly contractive mappings on partial metric spaces. These theorems generalize many previously obtained results. An example is given to show that our generalization from metric spaces to partial metric spaces is real.

1. Introduction and preliminaries

The notion of a partial metric space (PMS) was introduced in 1992 by Matthews [1,2]. The PMS is a generalization of the usual metric space in which the $d(x, x)$ are no longer necessarily zero. Recently, many authors have focused on partial metric spaces and their topological properties (see e.g. [3–6]).

A partial metric space (see e.g. [1,2]) is a pair $(X, p : X \times X \to \mathbb{R}^+)$ (where $\mathbb{R}^+$ denotes the set of all non-negative real numbers) such that:

(PM1) $p(x, y) = p(y, x)$ (symmetry);
(PM2) if $0 \leq p(x, x) = p(x, y) = p(y, y)$ then $x = y$ (equality);
(PM3) $p(x, x) \leq p(x, y)$ (small self-distances);
(PM4) $p(x, z) + p(y, y) \leq p(x, y) + p(y, z)$ (triangularity);

for all $x, y, z \in X$.

For a partial metric p on X, the function $d_p : X \times X \to \mathbb{R}^+$ given by

$$d_p(x, y) = 2p(x, y) - p(x, x) - p(y, y) \tag{1.1}$$

is a (usual) metric on X. Each partial metric p on X generates a T_0 topology τ_p on X with a base of the family of open p-balls $\{B_p(x, \varepsilon) : x \in X, \varepsilon > 0\}$, where $B_p(x, \varepsilon) = \{y \in X : p(x, y) < p(x, x) + \varepsilon\}$ for all $x \in X$ and $\varepsilon > 0$.

Definition 1 (*See e.g. [1,2,6]*).

(i) A sequence $\{x_n\}$ in a PMS (X, p) converges to $x \in X$ if and only if $p(x, x) = \lim_{n\to\infty} p(x, x_n)$.

E-mail address: thabet@cankaya.edu.tr.

0895-7177/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.mcm.2011.07.013