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Fixed point theorems for nonlinear weakly C-contractive mappings in metric spaces

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ABSTRACT

The purpose of this paper is to present some fixed point and coupled fixed point theorems for a nonlinear weakly C-contraction type mapping in metric and ordered metric spaces. Also, an example is given to support our results. Our results generalize several well-known results from the current literature.

1. Introduction

The Banach contraction mapping principle [1] is a very popular tool for solving the existence of problems in many branches of mathematical analysis. Generalizations of this principle have been established in various settings (see [2–14]). Chatterjea [15] introduced the concept of C-contraction as follows.

Definition 1.1 (*[15]*)**.** A mapping $T : X \to X$ where (X, d) is a metric space is said to be a C-contraction if there exists $\alpha \in \left(0, \frac{1}{2}\right)$ such that for all $x, y \in X$, the following inequality holds:

$$d(Tx, Ty) \le \alpha(d(x, Ty) + d(y, Tx)).$$

Alber and Guerre-Delabriere [16] introduced the definition of weak ϕ-contraction.

Definition 1.2 (*[16]*)**.** A self mapping T on a metric space X is called weak ϕ-contraction if there exists a continuous nondecreasing function $\phi : [0, +\infty) \to [0, +\infty)$ with $\phi(t) = 0$ if and only if $t = 0$ such that

$$d(Tx, Ty) \le d(x, y) - \phi(d(x, y))$$

for each $x, y \in X$.

The notion of ϕ-contraction and weak ϕ-contraction has been studied by many authors (see [17,18,5–7,11,19,13,20]).

In recent years, many results have appeared related to fixed points in a complete ordered metric space (see, for example, [21–23,12,24,8,25–27]).

Chatterjea [15] proved that if X is complete, then every C-contraction has a unique fixed point. Choudhury [28] generalized the concept of C-contraction as follows.

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