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State-space methods for calculating concentration dynamics in multizone buildings

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ABSTRACT

This work explores analytic solutions to contaminant transport in multizone systems as a means of solving concentration dynamics more rapidly and providing insight into system behaviour. A general formulation is developed that is consistent with state-space theory. This is used to provide general and specific solutions for the concentration time series. In particular, an analytical expression is presented for the case of constant system parameters and constant input conditions using the eigenvalues and eigenvectors for the case of a diagonalisable state matrix for the multizone system. A method for using this expression to solve concentrations over a series of varying building and input states is developed, with potential for use as a rapid means of calculating concentrations at any time. In principle, this method could be used as a complementary method for solving concentration dynamics within multizone software. Analytical expressions for cumulative exposure for the same case are also presented. The characteristic behaviour of the solutions and their dependence on the eigenvalues and eigenvectors of the state matrix are explored for the steady state case and the decay solution. In particular, the behaviour in the late decay phase is shown to be characterised by a single decay rate, given by the smallest magnitude eigenvalue, and a fixed concentration ratio, given by the associated eigenvector. The independence of the concentration ratio in this phase to the initial conditions is also demonstrated. Two example cases are used to illustrate these and additional features.

1. Introduction

1.1. Motivation

There is a need to predict the transport of airborne contaminants within buildings for applications such as mitigating against chemical, biological and radiological [CBR] attack and for understanding and improving indoor air quality. Multizone models [1,2] and computational fluid dynamics (CFD) [3,4] are widely used for such studies and they can readily be applied to examine specific cases. However, CFD studies are limited by their speed of set up and execution for whole building studies, typically limiting their use to a relatively small set of scenarios. Multizone models are, in general, much faster to execute than CFD simulations and can be used where the well-mixed assumption is reasonable.

Multizone models, such as CONTAM [5] and COMIS [6,7], provide useful numerical tools to predict airflows in buildings and the resulting contaminant transport. However, for some applications, such as the optimisation of sensor placement [8] or source term estimation [9,10], it may be necessary to simulate many different contaminant transport scenarios. The computation of the concentration dynamics for these simulations can be very time consuming. Also, using these methods to gain insight into the concentration dynamics under a range of conditions is difficult without performing many separate calculations.

This paper explores an analytical approach to the solution of contaminant transport systems as an alternative to an iterative numerical method, building on previous work by Sinden [11], Sandberg [12], Godfrey [13], Jacquez [14,15], Axley [16], Evans [17] and Sherman [18] amongst others. In particular we use a state-space formulation and explore the importance of eigenvalue solutions to the equation systems. A framework for an analytically based solution method for concentration dynamics is presented that may have considerable advantages for subclasses of multizone problems. Analytical expressions for cumulative exposure to contaminants are presented. The insights into the system behaviour provided by this method are also explored.

1.2. Background

Sinden [11] developed a multi-chamber theory of tracer transport in a ventilated building. He used matrix notation to write

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