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New parametric prioritization methods for an analytical hierarchy process based on a pairwise comparison matrix

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ABSTRACT

In this paper, new parametric prioritization methods (PPMs) to determine a family of priority vectors in an analytical hierarchy process (AHP) are proposed, pointing out the logical relation of elements in the comparative matrix. The scales and consistency cannot determine the priorities, but only the order of the alternatives. To derive the priorities of alternatives, a series of theorems and mathematical programming models is given based on a pairwise comparison matrix. This refers to parameters θ, α, β, by which there exists a family of priorities for the same judgment matrix. The discrimination of alternatives can be easily improved when using the proposed priority method by modifying the values of the parameters. Some false cognitions about how to determine the priority of an analytical hierarchy process are rectified. One should not elicit priority vectors from the judgment matrix; the information is incomplete, and the parameters must be considered. Finally, the meanings of parameters are explained in practical applications and an approach for determining the values of the parameters is proposed. Examples are also used to illustrate the features and applicability of the new approach in an AHP.

1. Introduction

As the analytic hierarchic process (AHP) [1–3] is developing and it is being used more and more widely in practice, it appears to be a popular tool for decision support [4]. The estimation of priorities from pairwise comparison matrices is the central part of the AHP. Generally, pairwise comparison matrices have two forms: the complementary judgment matrix and the reciprocal judgment matrix [5]. For the former, there exist mainly three methods to deal with the priorities of alternatives based on the complementary judgement matrices [6]: (1) transforming the complementary pairwise comparison matrices into consistent ones and then using the square method to obtain the priorities [7]; (2) transforming the complementary judgement matrices into consistent ones and then using the normalizing rank aggregation method to get the priorities [8,9]; and (3) using the formula $\varepsilon(W_i - W_j)$ to approximate the elements r_{ij} in the complementary judgement matrices $R = (r_{ij})_{n\times n}$, and then adopting the minimum two multiplication method to calculate the priorities [10]. Additionally, Zhang [6] proposed a theorem to present a priority method and showed the advantages by comparing his method with the one in the literature [11]. For reciprocal judgment matrices, the eigenvector (EV) method was first proposed by Saaty [1], who proved that the principal eigenvector of the comparison matrix can be used as a required priority vector, both for consistent and inconsistent judgments of the decision maker (DM). A standard procedure for extracting this vector by the EV method is based on consecutive squaring of the comparison matrix and normalizing row sums. Saaty also suggested several simple approximate methods to obtain the required priority vector. The simplest one may be referred to as the additive normalization (AN) method; it derives priorities by taking the sums of columns in the comparison matrix and by

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