Mathematical and Computer Modelling 54 (2011) 2616–2622

Contents lists available at SciVerse ScienceDirect

Mathematical and Computer Modelling

journal homepage: www.elsevier.com/locate/mcm

Continuous spectra and numerical eigenvalues

Oksana Guba [a,*], Jens Lorenz [b]

[a] Sandia National Laboratories,[1] PO Box 5800, Albuquerque, NM 87185-1322, United States

[b] Department of Mathematics and Statistics, University of New Mexico, Albuquerque, NM 87131, United States

ARTICLE INFO

Article history:
Received 14 June 2011
Accepted 14 June 2011

Dedicated to Heinz-Otto Kreiss' 80th Birthday

Keywords:
Eigenvalue problems
Continuous spectrum
QR Algorithm

ABSTRACT

Some spectral problems for differential operators are naturally posed on the whole real line, often leading to eigenvalues plus continuous spectrum. Then the numerical approximation typically involves three processes: (a) reduction to a finite interval; (b) discretization; (c) application of a numerical eigenvalue solver such as the QR-algorithm.

Reduction to a finite interval and discretization typically eliminate the continuous spectrum. However, through round-off error, the continuous spectrum may show up again when the eigenvalue solver is applied. (In some sense, three wrongs make a right.) Interestingly, not all parts of the continuous spectrum show up in the same way, however. We illustrate this observation by numerical examples. A perturbation argument, though non-rigorous, explains the observation.

1. On numerical spectra for the linearized Burgers' equation

The stability of a traveling wave depends on the spectrum of a differential operator L obtained by linearization about the wave profile. As a simple example, consider Burgers' equation

$$u_t = u_{xx} - \frac{1}{2}(u^2)_x, \quad x \in \mathbb{R},\ t \geq 0,$$

with stationary solution $U(x) = -\tanh \frac{x}{2}$. Linearization about $U(x)$ leads to the spectral problem

$$Lu \equiv u_{xx} - (Uu)_x = su \quad \text{where } L : H_2(\mathbb{R}) \to L_2(\mathbb{R}). \tag{1}$$

In this case, the operator L has the simple eigenvalue $s_0 = 0$ with corresponding eigenfunction $u_0(x) = U'(x)$. Also, since $U(x) \to \pm 1$ as $x \to \mp\infty$, the operator L has the same continuous spectrum as the operators $L_+u = u_{xx} + u_x$ and $L_-u = u_{xx} - u_x$. Therefore, the continuous spectrum of L is the parabolic line

$$\sigma_{cont} = \{s \in \mathbb{C} \mid s = -k^2 + ik,\ k \in \mathbb{R}\} \tag{2}$$

obtained by applying $L_\pm$ to $u(x) = e^{ikx}$.

Note that L_+ and L_- both have the same continuous spectrum, σ_{cont}, given in (2). Thus, for the operator L in (1) the line (2) should be thought of as double. In the next section we modify Burgers' equation to break the double line into two distinct parabolas. Doing this in two different ways, will more clearly illustrate the main point of the paper.

* Corresponding author.
E-mail addresses: onguba@sandia.gov (O. Guba), lorenz@math.unm.edu (J. Lorenz).

[1] Sandia National Laboratories is a multi-program laboratory operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the US Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

0895-7177/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.mcm.2011.06.037