



Uncertainty propagation or box propagation

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ABSTRACT

This paper discusses the use of recently developed techniques and software in the numerical propagation of uncertainties in initial coordinates and/or parameters for initial value problems. We present an approach based on several validated numerical integration techniques but focusing on the propagation of boxes. The procedure uses a multivariable high order Taylor series development of the solution of the system whose Taylor coefficients are calculated via extended automatic differentiation rules for all the basic operations. These techniques are implemented in the recent free-software TIDES. The classical two-body and Lorenz problems are chosen as examples to show the benefits of the approach. The results show that the solution of uncertainties can be approximated in an analytic form by means of a Taylor series and that these techniques can be extremely useful in different practical applications.

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1. Introduction

Initial value problems arise in a great deal of practical situations, and in many cases there are parameters in the definition of the system. When the differential equations come from a real setting, usually there are uncertainties in the values of the parameters and/or in the initial conditions, due to measurement limitations or due to mistakes. Therefore, in many engineering and physical studies it is typical to measure, in some sense, the influence of these possible differences with the real value. A typical approach in engineering is to use a linearized model or a full nonlinear Monte Carlo simulations, but also to use a validated numerical ODE software to give a rigorous solution that takes into account all the uncertainties. The first method, the use of a linearized model, lacks of precision and the Monte Carlo simulations are usually quite time consuming. The use of validated numerical ODE software, as VNODE [1], can lead to a wrapping problem, giving a large overestimation of the uncertainties. Other validated numerical ODE softwares are CAPD [2,3], oriented for Computer Assisted Proofs, and COSY INFINITY [4] that uses, instead of standard interval ODE methods, Taylor models [5] and interval arithmetic. Related with this approach, in [6] it is proposed to use high order multivariable Taylor series approximations in Astrodynamics, but the computation of the coefficients is done using the high order variational equations, being impossible (or at least too cumbersome) to go further than order 4 in the development, and in [7], also in Astrodynamics, it is used directly, the Taylor models of Berz.

In this paper we take the idea of the Taylor models but without the interval computation. This enables us to compute in a fast and simple way the multivariable Taylor development of the solutions, and so the solution uncertainties are approximated in an analytic form. Also, we use extended automatic differentiation (AD) rules for the direct computation of the Taylor coefficients of the multivariable polynomial (hence, in a completely different way as in COSY INFINITY) that permit us to focus just on the partial derivatives we are interested on.

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