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Some applications of Burzyński yield condition in metal plasticity

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ABSTRACT

The classical J_2 plasticity theory is widely used to describe the plastic response of metallic materials. However, this theory does not provide satisfactory predictions for materials which exhibit pressure-sensitive yielding or plastic dilatancy. Another difficulty is the difference between the values of yield stresses in tension and compression for isotropic materials, the so-called strength differential effect (SD), leading to the asymmetry of the elastic range. The Burzyński yield condition, proposed in 1928, can be used to overcome some of these problems. In this paper an implicit integration of the elasto-plastic constitutive equations for the paraboloid case of Burzyński's yield condition is formulated. Also, the tangent operator consistent with the integration algorithm was developed and is presented. The proposed model was implemented in a commercial Finite Element code and different kinds of tests reported in the literature were simulated. The comparison between the numerical and experimental results shows that the plasticity theory with the paraboloid case of Burzyński's yield condition describes adequately the strength differential effect, which is present in many kinds of materials significant for recent applications.

1. Introduction

The analysis of complex structural components, their design optimization and structural reliability assessment require the use of proper and accurate constitutive models to describe a material behaviour. To date, an overwhelming majority of structural analysis employ the classical J_2 plasticity theory to describe the plastic response of metallic alloys. This theory assumes that hydrostatic stress has no effect on plastic flow and the material is incompressible in the plastic regime.

However, there are reported in the literature on experimental results for metallic solids under quasi-static conditions, which exhibit pressure-sensitive yielding and plastic dilatancy and reveal inconsistency of the model based on the Huber–Mises yield condition [1,2]. In particular, this effect can be significant in designing, structural elements or machine parts in which stress concentrations may appear resulting in the increase of the value of the first stress invariant.

Another difficulty to be taken into account is the difference of the values of yield stresses in tension and compression for isotropic materials, the so-called strength differential effect, leading to the asymmetry of the elastic range.

The observed hydrostatic stress effect on the yield behaviour of the investigated metallic materials is commonly described in the literature by means of the adaptation of the criterion known in the soil mechanics in which the linear dependence of the yield limit on hydrostatic stress is assumed. Such a criterion was proposed originally in 1928 by Burzyński [3] and repeated later by other authors, e.g. by Drucker and Prager [4,5]. Recently, this kind of criterion has been used to model the pressure-sensitive yielding of metals [6–8]. However, it is known that the criterion represented by a conical failure surface in the space of principal stresses can only roughly approximate real behaviour of a material in the limited range of hydrostatic stress and fails to describe properly the states near to the apex of the failure cone [9].

In this paper the general Burzyński yield condition is reviewed showing that the Huber–Mises, Drucker–Prager and paraboloid Burzyński–Torre conditions can be received as particular cases of this more general model.

Then, an implicit integration of the elastoplastic constitutive equations for the paraboloid Burzyński yield condition is formulated. Also, the tangent operator consistent with the integration algorithm was developed and is presented. The proposed model was implemented in the Finite Element code *ABAQUS* through the user subroutine *UMAT*. This is a new contribution, which can find wide applications in practical analysis of complex states of stress in plasticity as well as viscoplasticity of metals and metal matrix composites [10]. In the latter paper an identical paraboloid yield criterion was considered. The authors related it with the names of Mises and Schleicher ([11] for a more detailed discussion of the historical background of the development of the paraboloid failure criteria). However, Zhang et al. [10] do not provide an integration algorithm of the developed equations of plasticity with the

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