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The modified couple stress functionally graded Timoshenko beam formulation

M. Asghari *, M. Rahaeifard, M.H. Kahrobaiyan, M.T. Ahmadian

School of Mechanical Engineering, Sharif University of Technology, Tehran, Iran

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ABSTRACT

In this paper, a size-dependent formulation is presented for Timoshenko beams made of a functionally graded material (FGM). The formulation is developed on the basis of the modified couple stress theory. The modified couple stress theory is a non-classic continuum theory capable to capture the small-scale size effects in the mechanical behavior of structures. The beam properties are assumed to vary through the thickness of the beam. The governing differential equations of motion are derived for the proposed modified couple-stress FG Timoshenko beam. The generally valid closed-form analytic expressions are obtained for the static response parameters. As case studies, the static and free vibration of the new model are respectively investigated for FG cantilever and FG simply supported beams in which properties are varying according to a power law. The results indicate that modeling beams on the basis of the couple stress theory causes more stiffness than modeling based on the classical continuum theory, such that for beams with small thickness, a significant difference between the results of these two theories is observed.

1. Introduction

Functionally graded materials (FGMs) are produced from mixing of two different materials. This type of materials provides the specific benefits of both of the constituents. They can be defined as inhomogeneous composites which are made from a mixture of two different materials, usually a metal and a ceramic, with a desired continuous variation of properties as a function of position along certain dimension(s). The continuously compositional variation of the constituents in FGMs along different directions is the great benefit of FGMs, because this property offers a solution to the problem of appearing high magnitude shear stresses that may be induced in laminated composites, where two materials with great differences in properties are bonded. Nowadays, structures made of FGMs have a great practical role in engineering and industrial fields.

Some works have been performed by researchers on the static and dynamic behavior of beams and plates made of FGMs. Asghari et al. [1] have mentioned some instances of these works, including Refs. [2–7]. As another instance, the thermal snapping of functionally graded plates has been investigated by Prakash et al. [8]. Also, Jomehzadeh et al. [9] presented an analytical approach for the stress analysis of functionally graded annular sector plates. Moreover, analytical modeling of thermal residual stresses in some functionally graded material systems has been presented by Bouchafa et al. [10]. It is noted that these sample works are based on the classical continuum theory, while the formulation presented in this work is based on a non-classical continuum theory, the modified couple stress theory, which is discussed in detail in the following.

In recent years, the application of FG materials has broadly been spread in micro and nano structures such as thin films in the form of shape memory alloys [11,12], micro- and nano-electromechanical systems (MEMS and NEMS) [13,14] and also atomic force microscopes (AFMs) [15]. Beams used in MEMS, NEMS and AFMs, have the thickness in the order of microns and sub-microns, so that the small scale effects in their behavior is considerable. The size-dependent static and vibration behavior in micro scales are experimentally validated (see for example [16–19]). Considering experimental observations, it is well-known that size-dependent behavior is an inherent property of materials which appears for a beam when the characteristic size such as thickness or diameter is close to the internal material length scale parameter [20].

The classical continuum mechanics theories are not capable of prediction and explanation of the size-dependent behaviors which occur in micron- and sub-micron-scale structures. However, non-classical continuum theories such as higher-order gradient theories and the couple stress theory are acceptably able to interpret the size-dependencies.

In 1960s some researchers introduced the couple stress elasticity theory [21–23]. In the constitutive equation of this theory, some higher-order material length scale parameters appear in addition to the two classical Lame constants. Yang et al. [24] argued that in addition to the classical equilibrium equations of forces and moments of forces, another equilibrium equation should be considered for the material elements. This additional equation is

* Corresponding author. Tel.: +98 21 66165523; fax: +98 21 66000021.
E-mail address: asghari@sharif.edu (M. Asghari).

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