Materials and Design 32 (2011) 5038–5042

Contents lists available at ScienceDirect

Materials and Design

journal homepage: www.elsevier.com/locate/matdes

Short Communication

An analogy between radially-loaded rubber bush mountings and axially-loaded bonded rubber blocks

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ARTICLE INFO

Article history:
Received 16 February 2011
Accepted 25 May 2011
Available online 1 July 2011

ABSTRACT

The validity of the widely-quoted experimental data for the reduced radial stiffness of a cylindrical rubber bush mounting of finite length is discussed by proposing an analogy between such a bush and two tandem cylindrical rubber blocks of circular cross-section which are axially loaded. Previously-derived expressions for the radial stiffness of a finite-length cylindrical bush and the apparent Young's modulus of an axially-loaded bonded rubber block of circular cross-section are exploited. This leads to an adaptation of the data that incorporates the possible failure of the bushes due to internal rupture. The analytically-derived values of this stiffness are demonstrated to have improved agreement with these appropriately modified data. Representative numerical results are presented for a range of bushes in tabular and graphical forms.

1. Introduction

A comprehensive range of rubber bush mountings is manufactured worldwide by diverse companies for installation in an extensive variety of modern components throughout the engineering industry. At the design and development stages, it is thus extremely important that readily-calculable expressions for predicting their stiffness under specified loading situations are available.

Adkins and Gent [1] presented widely-quoted experimental measurements, and a limited approximate theoretical treatment, of the deformations of a series of annular rubber bushes bonded to rigid metallic inner and outer cylindrical sleeves for radial, axial, torsional and tilting modes of deflection.

For the purposes of seeking a formula for the radial stiffness of a cylindrical bush which a practising design engineer could readily apply, many investigations have realistically taken the rubber to be homogeneous, isotropic and incompressible with the displacement gradients sufficiently small for the classical theory of elasticity to be applicable. Even with this model, it was over 45 years after Adkins and Gent performed their experiments before Horton et al. [2] used a superposition principle to derive an exact closed-form expression for the radial stiffness of a finite-length cylindrical bush. They presented and discussed the comparisons with the experimental values of Adkins and Gent [1] alongside the limited theoretical investigations of Hill [3], Freakley and Payne [4], Morman and Pan [5] and MRPRA [6].

Subsequently, this expression for the radial stiffness [2] has underpinned the development of models for more complex bush mountings. For example, it is inherent in a series of papers by Kari [7], Coja and Kari [8] and Tarrago et al. [9,10] that are motivated by analyzing noise abatement in vehicle suspension systems and pivot arms. Kari and his co-workers investigated the dynamic stiffness of rubber bushing in the audible-frequency domain, including amplitude dependence. Reassuringly Kari [7] compared the low-frequency limits of his dynamic stiffness with the static stiffness formula of Horton et al. [2] and concluded that the deviations are "rather small" and constitute "a fully negligible error from an acoustical point of view". Moreover, the related computational studies of Gough [11] and Kadlowec et al. [12] also incorporate the formula of Horton et al. [2] in analyzing the effects of swaging and nonlinearity, respectively, upon the stiffness.

Further, Horton and Tupholme [13] presented a derivation of a novel, more user-friendly, simple approximation to the exact closed-form formula given by Horton et al. [2] which could appeal to practising engineers. Gough [14] then reiterated the usefulness of such formulae when used alongside further refining predictions of finite element analysis for more complicated configurations.

Similarly, closed-form solutions were obtained by Horton et al. [15] for the deflection and stress distribution of axially-loaded bonded rubber blocks of circular (and also long-thin rectangular) cross-section. They demonstrated for the first time that the previously adopted assumption of parabolic deformed profiles, which was used by Gent and Lindley [16], is indeed invalid for small values of the shape factor, as originally speculated in 1959 by Hirst [17] and Payne [18]. Readily-calculable improved relations were deduced for the apparent Young's modulus.

The utility of their formula [15] for a circular block was first subsequently appreciated by Anderson et al. [19], when they enhanced its credence by remarking that it is the model which "offers the best

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doi:10.1016/j.matdes.2011.05.044