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Isogeometric solution of second order ordinary differential equations

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Abstract

In this research, based on the concept of isogeometric analysis, an algorithm is developed for solving second order ordinary differential equations. In isogeometric analysis method, solution of differential equations are considered as imaginary curves or surfaces which are constructed by using advanced versions of splines such as Non-Uniform Rational B-Splines (NURBS). In the same manner, coefficients of differential equations, which themselves might be functions in general, can be assumed as other imaginary curves or surfaces. An IGA framework is created in MATLAB to solve problems. The analysis result is compared with exact and other methods solutions. Finally, the effect of different parameters on the solution of an example is investigated.

Keywords: Isogeometric analysis, NURBS, second order ordinary differential equations

١. INTRODUCTION

Most of problems faced in different disciplines of science and engineering are engaged with solving differential equations. Since only a very limited of these equations can be solved analytically, several numerical methods have been proposed for the solution of differential equations. Amongst the most popular of these methods the finite difference, finite element and the wide range of so called mesh-free methods can be mentioned. The NURBS (non-uniform rational B-splines) based isogeometric analysis method proposed by Hughes et al. in ٢٠٠٥, removes some difficulties of existent methods such as requiring a mesh generation process and inaccurate modeling of the geometry [١].

Isogeometric analysis (IGA) is designed to combine two tasks, design by Computer Aided Design (CAD) and Finite Element Analysis (FEA). Isogeometric analysis is indeed a collection of methods that use splines, or some of their extensions such as NURBS and T-splines, as functions to build approximation spaces which are then used to solve partial differential equations numerically. Due to some interesting properties of splines and NURBS beside accurate definition of geometry, their basis functions can be employed in place of interpolation and approximation functions of finite elements and meshfree methods. From the standpoint of numerical solution of equations, the use of splines and NURBS as building blocks for the construction of discrete spaces, paves the way to many new numerical schemes for the numerical simulations of equations that would be extremely hard to achieve within a standard finite element framework. The smoothness of splines is a new ingredient that yields several advantages: for example, it improves the accuracy per degree of freedom and allows for the direct approximation of equations of order higher than two.

In this paper, based on the concept of isogeometrical analysis, an algorithm is developed for solving second order ordinary differential equations. In this case, the solution might be imagined as a curve which can be generated by using Splines and NURBS. In Section ٢, the main concepts of curve definition by Splines is briefly explained. Section ٣ is devoted to the derivation of the formulation and the system of equations. In Section ٤ the effect of different parameters on the solution of a typical example is investigated. Finally, Conclusions and proposed further research is the subject of Section ٥.