A new approach for numerical solution of Fokker-Plank Equation using the Chelyshkov cardinal Function

Davod Rostamy Nazdar Abdollahi * Samaneh Qasemi
Imam Khomeini International University

Abstract

In this paper a numerical method is presented for the solution of Fokker-Plank equation. The main idea of this method is expanding the approximate solution by the Chelyshkov cardinal function. At the end, using the operator derivative matrix the problem turns into a system of algebraic equations.

Keywords: Fokker-Plank equation, Chelyshkov polynomials, cardinal functions.

1 Introduction

In statistical mechanics, the Fokker-Plank is a partial differential equation that describe the time volution of the probability density function of the velocity of a particle under the influence of day forces and random forces as in Brownian motion. Fokker-Plank equation occurs in many different fields such as solid state physics, quantum optics, theoretical biology, ect. The general Fokker-Plank equation for the motion of a concentration field $u(x, t)$ of one space variable x at time t has the form

$$\frac{\partial u}{\partial t} = [-\frac{\partial}{\partial x}A(x,t) + \frac{\partial^2}{\partial x^2}B(x,t)]u. \tag{1}$$

$$u(x, , 0) = f(x) \qquad x \in (-\infty, \infty) \tag{2}$$

where $u(x, t)$ is unknown, $B(x, t) > 0$ is the diffusion coefficient and $A(x, t) > 0$ is the drift coefficient. We know that in one variable case the nonlinear Fokker-Plank equation is

$$\frac{\partial u}{\partial t} = [-\frac{\partial}{\partial x}A(x,t,u) + \frac{\partial^2}{\partial x^2}B(x,t,u)]u. \tag{3}$$

These polynomials are orthogonal respect to weight function 1 over the interval $[0, 1]$. The explicit definition of these polynomials are as follow

$$P_{N,k}(x) = \sum_{j=0}^{N-k}(-1)^j \left(\begin{array}{c} N-k \\ j \end{array} \right) \left(\begin{array}{c} N-k+1+j \\ N-k \end{array} \right) x^{k+j}, \qquad k = 0, 1, ..., N. \tag{4}$$

In the present paper, we consider these polynomials for the case $k = 0$, then (4) becomes to

$$P_{N,0}(x) = \sum_{j=0}^{N}(-1)^j \left(\begin{array}{c} N \\ j \end{array} \right) \left(\begin{array}{c} N+1+j \\ N \end{array} \right) x^{j}, \tag{5}$$

*Speaker