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Uniformly local biorthogonal wavelet constructions on intervals by extension operators

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Abstract

We construct a basis for a range of Sobolev spaces on interval $(-1,1)$ from corresponding bases on $(-1,0)$ and $(0,1)$ by the application of extension operators. Two examples of Hestenes extensions (as extension operators) are presented for constructing wavelets that are in $C^0(-1,1)$ and $C^1(-1,1)$.

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1 Introduction

For $t \in [0,\infty) \setminus (\mathbb{N}_0 + \{\frac{1}{2}\})$ and $\vec{\sigma} = (\sigma_\ell, \sigma_r) \in \{0, \ldots, \lfloor t + \frac{1}{2} \rfloor\}^2$, let

$$H^t_{\vec{\sigma}}(\mathcal{I}) := \{v \in H^t(\mathcal{I}) : v(0) = \cdots = v^{(\sigma_\ell - 1)}(0) = 0 = v(1) = \cdots = v^{(\sigma_r - 1)}(1)\}.$$

For t and $\vec{\sigma}$ as above, and for $\tilde{t} \in [0,\infty) \setminus (\mathbb{N}_0 + \{\frac{1}{2}\})$ and $\vec{\tilde{\sigma}} = (\tilde{\sigma}_\ell, \tilde{\sigma}_r) \in \{0, \ldots, \lfloor \tilde{t} + \frac{1}{2} \rfloor\}^2$, let *univariate wavelet* collections $\Psi_{\vec{\sigma},\vec{\tilde{\sigma}}} := \{\psi_\lambda^{(\vec{\sigma},\vec{\tilde{\sigma}})} : \lambda \in \nabla_{\vec{\sigma},\vec{\tilde{\sigma}}}\}$, $\tilde{\Psi}_{\vec{\sigma},\,\vec{\tilde{\sigma}}} := \{\tilde{\psi}_\lambda^{(\vec{\sigma},\vec{\tilde{\sigma}})} : \lambda \in \nabla_{\vec{\sigma},\vec{\tilde{\sigma}}}\}$ be Riesz bases for $H^t_{\vec{\sigma}}(\mathcal{I})$ and $H^{\tilde{t}}_{\vec{\tilde{\sigma}}}(\mathcal{I})$, after renormalizing, that satisfy some properties in [1]. We assume to have available a univariate extension operator

$$\breve{G}_1 \in B(L_2(0,1), L_2(-1,1)) \text{ with } \begin{cases} \breve{G}_1 \in B(H^t(0,1), H^t(-1,1)), \\ \breve{G}_1^* \in B(H^{\tilde{t}}(-1,1), H^{\tilde{t}}_{(\lfloor \tilde{t}+\frac{1}{2}\rfloor, 0)}(0,1)). \end{cases} \tag{1}$$

Let η_1 and η_2 denote the extensions by zero of functions on $(0,1)$ and on $(-1,0)$ to functions on $(-1,1)$, respectively, with R_1 and R_2 denoting their adjoints. We assume that $\breve{G}_1$ and its "adjoint extension", i. e., $\breve{G}_2 := (\mathrm{Id} - \eta_1 \breve{G}_1^*)\eta_2$ are local. For $\breve{G}_1$, we will consider the Hestenes extension which is of the form $\breve{G}_1 v(-x) = \sum_{l=0}^{L} \gamma_l (\zeta v)(\beta_l x) \quad (v \in L_2(\mathcal{I}),\, x \in \mathcal{I})$, where $\gamma_l \in \mathbb{R}$, $\beta_l > 0$, and $\zeta : [0,\infty) \to [0,\infty)$ is a smooth cut-off function. Its adjoint reads as $\breve{G}_1^* w(x) = w(x) + \zeta(x) \sum_{l=0}^{L} \frac{\gamma_l}{\beta_l} w\left(\frac{-x}{\beta_l}\right)$ where $w \in L_2(-1,1)$ and $x \in \mathcal{I}$. A Hestenes extension satisfies (1) if and only if

$$\sum_{l=0}^{L} \gamma_l \beta_l^i = (-1)^i \, (\mathbb{N}_0 \ni i \leq \lfloor t - \tfrac{1}{2} \rfloor), \; \sum_{l=0}^{L} \gamma_l \beta_l^{-(j+1)} = (-1)^{j+1} \, (\mathbb{N}_0 \ni j \leq \lfloor \tilde{t} - \tfrac{1}{2} \rfloor).$$