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Domination polynomial of generalized friendship graphs

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Abstract

Let G be a simple graph of order n. The domination polynomial of G is the polynomial $D(G,x)=\sum_{i=0}^{n} d(G,i)x^i$, where $d(G,i)$ is the number of dominating sets of G of size i. Let n and $q\geq 3$ be any positive integer and $F_{q,n}$ be the *generalized friendship graph* formed by a collection of n cycles (all of order q), meeting at a common vertex. We study the domination polynomials of some generalized friendship graphs. In particular we examine the domination roots of these families, and find the limiting curve for the roots.

Keywords: Domination polynomial; friendship graph; flower graphs.
Mathematics Subject Classification [2010]: 05C60

1 Introduction

Let $G=(V,E)$ be a simple graph. For any vertex $v\in V(G)$, the *open neighborhood* of v is the set $N(v)=\{u\in V(G)|\{u,v\}\in E(G)\}$ and the *closed neighborhood* of v is the set $N[v]=N(v)\cup\{v\}$. For a set $S\subseteq V(G)$, the open neighborhood of S is $N(S)=\bigcup_{v\in S}N(v)$ and the closed neighborhood of S is $N[S]=N(S)\cup S$. A set $S\subseteq V(G)$ is a *dominating set* if $N[S]=V$ or equivalently, every vertex in $V(G)\backslash S$ is adjacent to at least one vertex in S. The *domination number* $\gamma(G)$ is the minimum cardinality of a dominating set in G. Let $\mathcal{D}(G,i)$ be the family of dominating sets of a graph G with cardinality i and let $d(G,i)=|\mathcal{D}(G,i)|$. The *domination polynomial* $D(G,x)$ of G is defined as $D(G,x)=\sum_{i=\gamma(G)}^{|V(G)|}d(G,i)x^i$, where $\gamma(G)$ is the domination number of G (see [1, 2]). A root of $D(G,x)$ is called a domination root of G. The set of distinct roots of $D(G,x)$ is denoted by $Z(D(G,x))$.

Calculating the domination polynomial of a graph G is difficult in general, as the smallest power of a non-zero term is the domination number $\gamma(G)$ of the graph, and determining whether $\gamma(G)\leq k$ is known to be NP-complete [6]. But for certain classes of graphs, we can find a closed form expression for the domination polynomial. The domination polynomial of friendship graphs and its limiting curve for their domination roots studied recently [3]. In this paper we consider *generalized friendship graph (or flower graphs)*, calculate their domination polynomials, exploring the nature and location of their roots.

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