On the Tangent Space of an $n-$Surface

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Abstract

We are supposed to characterize the tangent space of an $n-$surface $S = f^{-1}(c)$ for some $f : U \to R$, U open in R^{n+1}, and f is a smooth function with the property that $\nabla f(p) \neq 0$ for all $p \in S$, in the case that the whole space admits the general form of the inner product. Finally we introduce a vector field X with integral curve α through p such that the covariant derivative of f with respect to $\acute{\alpha}(0)$ at p, i.e., $\nabla_{\acute{\alpha}(0)} f$ has maximum value.

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1 Introduction

The idea of the definition of a regular surface is to introduce a set S, that is, in a certain sense, two dimensional and that also is smooth so that the usual notions of calculus can be extended to it [1, 9]. for example, if $x : U \subseteq R^2 \to S$ be a parameterization of a regular surface S and $q \in U$, then the vector subspace $dx_q R^2 \subseteq R^3$ of dimension 2, coincides with the set of tangent vectors of S at $x(q)$. In the case that $f : U \subseteq R^3 \to R$ is a smooth function and $a \in f(U)$ is a regular point of f, then $S = f^{-1}(a)$ is a regular surface in R^3. As a result, in this case, the tangent space of S at p consides with $\nabla f(p)^{\perp}$,i.e., the set of vectors at p which are perpendicular with respect to the usual inner product of R^3 to $\nabla f(p)$ [2, 10]. In this note we are supposed to characterized the tangent space T_pS for an $n-$surface S in the case that R^{n+1} admits a general inner product $\alpha_A(u, v) = uAv^t$, in which A is a symmetric positive definite $(n + 1) \times (n + 1)$ real matrix [3].

2 Preliminaries

Definition 2.1. [1] Let $f : U \to R$ be a smooth function, where $U \subseteq R^{n+1}$ is an open set, let $c \in R$ be such that $f^{-1}(c)$ is non-empty and let $p \in f^{-1}(c)$. A vector is said to be a tangent to the level set $f^{-1}(c)$ if it is the velocity vector of a parameterized curve in R^{n+1} whose image is contained in $f^{-1}(c)$.

Definition 2.2. A parameterized curve is a smooth function $\alpha : I \to R^{n+1}$ for some open interval I, and an $n-$surface is a non empty subset $S \subseteq R^{n+1}$ for some $n \in N$ of the form $S = f^{-1}(c)$ where $f : U \to R$, U open in R^{n+1}, is a smooth function with the property that $\nabla f(p) \neq 0$ for all $p \in S$.

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