Solving nonlinear fuzzy differential equations by the Adomian-Tau method

Tayebeh Aliabdoli Bidgoli *
Shahid Bahonar University of Kerman

Abstract

In this paper, a numerical method for nonlinear fuzzy differential equations is presented. The method is based on Adomian-Tau method. Numerical examples are presented to verify the efficiency and accuracy of the proposed method.

Keywords: fuzzy differential equation, generalized differentiable, Adomian-Tau method.
Mathematics Subject Classification [2010]: 34A07

1 preliminary

In this section,we present definitions and concepts that need in throughout papers.
Let us denote by $\mathbb{R}_{\mathcal{F}}$ the class of fuzzy subsets of the real axis $u : \mathbb{R} \to [0,1]$, such that u is normal, upper semicontinuous and convex fuzzy set with compact support. Then $\mathbb{R}_{\mathcal{F}}$ is called the space of fuzzy numbers. For $0 < \alpha \leq 1$, denote $[u]^\alpha = \{x \in \mathbb{R}; u(x) \geq \alpha\}$ and $[u]^0 = \{x \in \mathbb{R}; u(x) > 0\}$. Then it is well- known that for any $\alpha \in [0,1]$, $[u]^\alpha$ is a bounded closed interval. For $u, v \in \mathbb{R}_{\mathcal{F}}$, and $\lambda \in \mathbb{R}$, the sum $u + v$ and the product $\lambda.u$ are defined by $[u+v]^\alpha = [u]^\alpha + [v]^\alpha$, $[\lambda.u]^\alpha = \lambda[u]^\alpha, \forall \alpha \in [0,1]$, where $[u]^\alpha + [v]^\alpha = \{x + y : x \in [u]^\alpha, y \in [v]^\alpha\}$ means the usual addition of two intervals of $\mathbb{R}$ and $\lambda[u]^\alpha = \{\lambda x : x \in [u]^\alpha\}$ means the usual product between a scalar and a subset of $\mathbb{R}$.
Let $D : \mathbb{R}_{\mathcal{F}} \times \mathbb{R}_{\mathcal{F}} \to \mathbb{R}^+ \cup \{0\}$, $D(u,v) = \sup_{\alpha \in [0,1]} \max\{|\underline{u}^\alpha - \underline{v}^\alpha|, |\overline{u}^\alpha - \overline{v}^\alpha|\}$, be the Hausdorff distance between fuzzy numbers, where $[u]^\alpha = [\underline{u}^\alpha, \overline{u}^\alpha]$, $[v]^\alpha = [\underline{v}^\alpha, \overline{v}^\alpha]$. The following properties are well-known

- $D(u+w, v+w) = D(u,v), \quad \forall u, v, w \in \mathbb{R}_{\mathcal{F}}$,
- $D(k.u, k.v) = |k| D(u,v), \quad \forall k \in \mathbb{R}, u, v \in \mathbb{R}_{\mathcal{F}}$,
- $D(u+v, w+e) \leq D(u,w) + D(v,e), \forall u, v, w, e \in \mathbb{R}_{\mathcal{F}}$,

and $(\mathbb{R}_{\mathcal{F}}, D)$ is a complete metric space.

Definition 1.1. Let $x, y \in \mathbb{R}_{\mathcal{F}}$. If there exist $z \in \mathbb{R}_{\mathcal{F}}$ such that $x = y + z$, then z is called the $H-$ difference of x and y and it is denoted by $x \ominus y$.

In this paper the " $\ominus$ " sign stands always for $H-$ difference and let us remark that $x \ominus y \neq x + (-1)y$.

Definition 1.2. [1] Let $f : (a,b) \to \mathbb{R}_{\mathcal{F}}$ and $x_0 \in (a,b)$, then f is strongly generalized differential on x_0, if there exists an element $f'(x_0) \in \mathbb{R}_{\mathcal{F}}$, such that

*Speaker