Some new subgroupoids of topological fundamental groupoid

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Abstract

$\lim_{hh} lk$ In this talk, we introduce some subgroupoids of the fundamental groupoid of locally wild spaces by using the recently emerged subgroups of the fundamental group, $\pi_1^s(X,x)$, $\pi_1^{sg}(X,x)$ and $\pi_1^{sp}(X,x)$, for a given space X which is not semi-locally simply connected. Also, we use the advantages of covering groupoid theory to find categorical universal covering of these spaces.

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1 Introduction

A groupoid G is a small category in which each morphism is an isomorphism. In a groupoid G, we call morphisms as elements of G and for $x, y \in O(G) = objec(G)$ we write $G(x, y)$ for the set of all morphisms with initial point x and final point y. The object group at x is $G(x) = G(x, x)$. For $x \in O(G)$, by $star_G x$ we mean the set of all the elements of G such that initiate at x.

A morphism of groupoids $\widetilde{G}$ and G is a functor, i.e., it consists of a pair of functions $f : \widetilde{G} \longrightarrow G$, $O(f) : O(\widetilde{G}) \longrightarrow O(G)$ preserving all the structure. Let $f : \widetilde{G} \longrightarrow G$ be a morphism of groupoids. Then f is called a covering morphism if for each $\tilde{x} \in \widetilde{G}$, the restriction $star_{\widetilde{G}}\tilde{x} \longrightarrow star_G f(\tilde{x})$ of f is bijective.

Let G be a groupoid. A subgroupoid of G is a subcategory H of G such that $a \in H$ implies that $a^{-1} \in H$; that is, H is a subcategory which is also a groupoid. A subgroupoid N of G is called normal if N is wide in G (as a subcategory) and, for any objects x, y of G and a in $G(x, y)$, $aN(x)a^{-1} \subseteq N(y)$. If N is a normal subgroupoid of G such that $N(x, y) = \emptyset$ for $x \neq y$, the quotient groupoid of G by N is a groupoid G/N by object set as same as G and $G/N(x, y) = \{aN(x) \ : \ a \in G(x, y)\}$, for any $x, y \in Object(G/N)$ with the multiplication that if $a \in G(x, y)$ and $b \in G(y, z)$ then $bN(y)aN(x) = baN(x)$.

For a topological space X, the homotopy classes of the paths in X form a groupoid on X. The composition of paths in X induces a composition of the homotopy classes. This groupoid is called fundamental groupoid and denoted by $\pi_1 X$. (see [1])

When the space X is not semi-locally simply connected, some subgroups of the fundamental group will emerge that have important role in the classification of the categorical universal covering.

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