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Weak fixed point property in closed subspaces of some compact operator spaces

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Abstract

For suitable Banach spaces X and Y with Schauder decompositions and closed subspace M of some compact operator spaces from X to Y, it is shown that the complete continuity of all evaluation operators on M, is a sufficient condition for the weak fixed point property of M; where for each $y^* \in Y^*$, the evaluation operator on M is defined by $\psi_{y^*}(T) = T^* y^*$, $T \in M$.

Keywords: weak fixed point property, evaluation operator, compact operator, completely continuous operator

Mathematics Subject Classification [2010]: 47H10, 47L05

1 Introduction

If C is a subset of a Banach space X, a mapping $T : C \to X$ is called a nonexpansive map if $\|Tx - Ty\| \leq \|x - y\|$ for all $x, y \in C$. We say that X has the fixed point property (fpp) if every nonexpansive self map $T : C \to C$ of each nonempty, closed, bounded and convex subset C of X has a fixed point. But when the same holds for every nonempty weakly compact convex subset of X, we say that X has the weak fixed point property (wfpp). It is evident that fpp implies the wfpp and for reflexive Banach spaces, both properties are the same.

For example, every uniformly convex Banach space and every Banach space with uniform normal structure have the fpp [8], every Banach space with weak normal structure and every Banach space with the Schur property (i.e. the weak and norm convergence of sequences are the same), have the wfpp [12, 8].

Following the work of Maurey [10] and Dowling-Lennard [7], which proved that a closed subspace M of the Bochner integrable function space $L^1([0, 1])$, has the fpp if and only if M is reflexive; it is natural to ask for a given Banach space X, what closed subspaces of it have the (weak) fpp.

There are a few works on fpp and wfpp in operator spaces. In 1999, Dowling and Randrianantoanina [6] along with a result of Besbes [4], have shown that a closed subspace of $K(H)$, of all compact operators on the Hilbert space H, has the fixed point property if and only if it is reflexive. Also, the Banach space $K(l^2)$, and then all it's closed subspaces has the wfpp [4].

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