



# Hopf bifurcation in a general class of delayed BAM neural networks

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## Abstract

In this paper, Hopf bifurcation analysis of delayed BAM neural networks, which consist of one neuron in the X-layer and other neurons in the Y-layer, will be discussed. Here, the number of neurons can be chosen arbitrarily. The associated characteristic equation is studied by classification according to the number of neurons. Numerical examples are also presented.

**Keywords:** Hopf bifurcation, Time delay, Characteristic equation

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## 1 Introduction

Since Hopfield constructed a simplified neural network (NN) model [1], the dynamical characteristics of artificial neural networks have been applied in many sciences such as mathematics, physics and computer sciences. As time delays always occur in the signal transmission, Marcus and Westervelt proposed an NN model with delay [2].

The bidirectional associative memory (BAM) networks were first introduced by Kasko (see [3]). It is well known that BAM NNs are able to store multiple patterns, but most of NNs have only one storage pattern or memory pattern. BAM NNs have practical applications in storing paired patterns or memories and possess the ability of searching the desired patterns through both forward and backward directions. It should be noted that periodic solutions can be resulted from the Hopf bifurcation in delay differential equations. In fact, various local periodic solutions can arise from the different equilibrium points of BAM NNs by applying Hopf bifurcation technique.

The delayed BAM neural network is described as follows:

$$\begin{cases} \dot{x}_i(t) = -\mu_i x_i(t) + \sum_{j=1}^m c_{ji} f_i(y_j(t - \tau_{ji})) + I_i & (i = 1, 2, \dots, n) \\ \dot{y}_j(t) = -\nu_j y_j(t) + \sum_{i=1}^n d_{ij} g_j(x_i(t - \sigma_{ij})) + J_j & (j = 1, 2, \dots, m) \end{cases} \quad (1)$$

where  $c_{ji}$  and  $d_{ij}$  are the connection weights through the neurons in two layers: the X-layer and the Y-layer. The stability of internal neuron processes on the X-layer and Y-layer are described by  $\mu_i$  and  $\nu_j$ , respectively. On the X-layer, the neurons whose states are denoted by  $x_i(t)$  receive the input  $I_i$  and the inputs outputted by those neurons in the

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