Exist and uniqueness of p-best approximation in fuzzy normed spaces

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Abstract

In this paper, we define a fuzzy normed space and study the concept of p-best approximation in fuzzy normed spaces. We also define a p-proximal set and p-Chebyshev set and prove some interesting results in this newsetup.

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1 Introduction

In this section we recall some notations and basic definitions used in this paper. A function $f : \mathbb{R} \to \mathbb{R}_0^+ = [0,1]$ is called a distribution function if it is non-decreasing and left continuous with $\inf_{t\in\mathbb{R}} f(t) = 0$ and $sup_{t\in\mathbb{R}} f(t) = 1$. By D^+, we denote the set of all distribution functions such that $f(0) = 0$. If $a \in \mathbb{R}_0^+$, then $H_a \in D^+$, where

$$H_a(t) = \begin{cases} 1 & t > a, \\ \\ 0 & t \le a. \end{cases}$$

A t-norm is a continuous mapping $* : [0,1] \times [0,1] \to [0,1]$ such that $([0,1], *)$ is an abelian monoid with unit one and $a * b \le c * d$ if $a \le c$ and $b \le d$ for all $a, b, c \in [0,1]$.

Definition 1.1. Let X be a linear space of a dimension greater than one, $*$ a t-norm continuous, and let N be a mapping from $X \times \mathbb{R}$ into D^+. The following conditions are satisfied for all $x, y \in X$ and $t, s > 0$,
(i) $N(x;t) = H_0(t)$ if and only if $x = \theta$ (θ is the null vector in X),
(ii) $N(\alpha x;t) = N(x; \frac{t}{|\alpha|})$ for all t in $\mathbb{R}^+$,
(iii) $N(x + y; t + s) \ge N(x;t) * N(y;s)$.
Triple $(X, N, *)$ is called a fuzzy normed space. If in addition, $t > 0$, $(x) \to N(x;t)$ is a continuous map on X, then $(X, N, *)$ is called a strong fuzzy normed space.

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