46th Annual Iranian Mathematics Conference
25-28 August 2015
Yazd University
Talk On hypergroups with trivial fundamental group pp.: 1–4

On hypergroups with trivial fundamental group

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Abstract

Let $(H, \circ)$ be a hypergroup. Consider the fundamental relation β^*, as the smallest equivalence relation on H, such that the quotient algebraic structures $(H/\beta^*, \otimes)$, the fundamental group of H, is a group. In this paper we investigate some conditions such that for a given finite hypergroup H, its fundamental group $(H/\beta^*, \otimes)$ is a trivial group.

Keywords: OC-Hypergroup, Adapted Hypergroup, TS-Hypergroup, Identical Hypergroup, m^n-Hypergroup
Mathematics Subject Classification [2010]: 20N20

1 Introduction

The concept of hypersrtucture was defined by Marty in 1934 [6]. A non-empty set H together with a mapping $\circ$ (namely hyperproduct) from $H \times H$ into $P^*(H)$, the set of all non-empty subsets of H, is called a *hypergroupoid* and denoted by $(H, \circ)$. If there is no ambiguity, we simply write H instead of $(H, \circ)$. For two non-empty subsets $A, B \subseteq H$, define $A \circ B = \bigcup_{(a,b) \in A \times B} a \circ b$. By abuse of notation, $a \circ b = \{x\}$, $A \circ \{a\}$ and $\{a\} \circ A$ are denoted by $a \circ b = x$, $A \circ a$ and $a \circ A$, respectively. A hypergroupoid $(H, \circ)$ is called a *hypergroup* if $\circ$ is associative and $H \circ x = x \circ H = H$, for every $x \in H$ (*reproduction axiom*). From now on, if there is no ambiguity, by xy (for $x, y \in H$) and H, we mean $x \circ y$ and hypergroup $(H, \circ)$, respectively. A hypergroup H is *commutative* if $xy = yx$ for every $x, y \in H$. Many books and papers has been written about the applications of hyperstructures theory in mathemathics and even other sciences ([1, 2, 3]). The purpose of this paper is to study some finite hepergroups that have trivial fundamental group. In this regards, we introduce the notion of overlapped covering of a hypergrpup, which leads us to class of *OC-hypergroups*, and then some special subclasses, namely class of *adapted hypergroups* and class of *TS-hypergroups*. First, we need some general and basic concepts of hyperstructures theory.
A non-empty subset A of the hypergroup H is called a *complete part* of H if for all positive integer n and for all $(x_1, x_2, ..., x_n) \in H^n$, $\prod_{i=1}^n x_i \bigcap A \neq \emptyset$ implies $\prod_{i=1}^n x_i \subseteq A$. The complete closure of A in H is the intersection of all complete parts containing A and is denoted by $C(A)$ and is equal to $K(A)$ that is obtained as the following way:

$$K_1(A) = A,$$

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