The restrained k-rainbow reinforcement numbers in graphs

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Abstract

for a positive integer k, a *restrained k-rainbow dominating function* (RkRDF) of a graph G is a function f from the vertex set $V(G)$ to the set of all subsets of the set $\{1, 2, \ldots, k\}$ such that for any vertex $v \in V(G)$ with $f(v) = \emptyset$ the conditions $\bigcup_{u \in N(v)} f(u) = \{1, 2, \ldots, k\}$ and $|N(v) \cap \{u \in V(G) \mid f(u) = \emptyset\}| \geq 1$ are fulfilled, where $N(v)$ is the open neighborhood of v. The *weight* of an RkRDF f is the value $\omega(f) = \sum_{v \in V(G)} |f(v)|$. The *restrained k-rainbow domination number* of a graph G, denoted by $\gamma_{rrk}(G)$, is the minimum weight of an RkRDF of G. The restrained k-rainbow reinforcement number $r_{rrk}(G)$ of a graph G is the minimum number of edges that must be added to G in order to decrease the restrained k-rainbow domination number. In this paper, we initiate the study of restrained k-rainbow reinforcement number in graphs and we present some sharp bounds on $r_{rrk}(G)$. In particular, we determine the restrained 2-rainbow reinforcement number of some classes of graphs.

Keywords: Retrained k-rainbow domination number, restrained k-rainbow reinforcement number.

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1   Introduction

In this paper, G is a simple graph with vertex set $V = V(G)$ and edge set $E = E(G)$. The order $|V|$ of G is denoted by $n = n(G)$. For every vertex $v \in V(G)$, the *open neighborhood* $N_G(v) = N(v)$ is the set $\{u \in V(G) \mid uv \in E(G)\}$ and the *closed neighborhood* of v is the set $N_G[v] = N[v] = N(v) \cup \{v\}$. The *degree* of a vertex $v \in V$ is $\deg_G(v) = \deg(v) = |N(v)|$. The *minimum* and *maximum degree* of a graph G are denoted by $\delta = \delta(G)$ and $\Delta = \Delta(G)$, respectively. We write $K_{n,m}$ for the *complete bipartite graph* of order $n + m$, C_n for a *cycle* of length n and P_n for a path of order n.

A subset S of vertices of G is a *dominating set* if $N[S] = V$. The *domination number* $\gamma(G)$ is the minimum cardinality of a dominating set of G. A dominating set of minimum cardinality of G is called a $\gamma(G)$-set. The *reinforcement number* $r(G)$ of a graph G is the minimum number of edges that must be added to G in order to decrease the domination number [13]. The reinforcement number is defined to be 0 when $\gamma(G) = 1$.

For a positive integer k, a *restrained k-rainbow dominating function*(RkRDF) of a graph G is a function f from the vertex set $V(G)$ to the set of all subsets of the set $\{1, 2, \ldots, k\}$

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