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Homological properties of certain subspaces of $L^{\infty}(G)$ on group algebras

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Abstract

Homological properties of several Banach left $L^1(G)$-modules have been studied by Dales and Polyakov and recently by Ramsden. In this paper, we characterize homological properties for some sub-modules of $L^{\infty}(G)$ as Banach left $L^1(G)$-modules.

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1 Introduction

Throughout this paper, G denotes a locally compact group with the identity element e, the modular function Δ, and a fixed left Haar measure λ. As usual, let $L^1(G)$ denote the group algebra of G as defined in [4] equipped with the norm $\|\cdot\|_1$ and the convolution product $*$ of functions on G defined by

$$(\phi * \psi)(x) = \int_G \phi(y)\psi(y^{-1}x)\, d\lambda(y)$$

for all $\phi, \psi \in L^1(G)$ and locally almost all $x \in G$. Let also $L^{\infty}(G)$ denote the Banach space as defined in [4] equipped with the essential supremum norm $\|\cdot\|_{\infty}$. Then $L^{\infty}(G)$ is the dual bimodule of the Banach $L^1(G)$-bimodule $L^1(G)$ under the pairing

$$\langle f, \phi\rangle = \int_G f(x)\phi(x)\, d\lambda(x).$$

for all $\phi \in L^1(G)$ and $f \in L^{\infty}(G)$. The left and right module actions of $L^1(G)$ on $L^{\infty}(G)$ are given by the formulae

$$\phi \cdot f = f * \widetilde{\phi} \quad \text{and} \quad f \cdot \phi = \frac{1}{\Delta}\widetilde{\phi} * f$$

for all $f \in L^{\infty}(G)$ and $\phi \in L^1(G)$, where $\widetilde{\phi}(x) = \phi(x^{-1})$ for all $x \in G$. We denote by $C_b(G)$ the space of all bounded continuous functions on G, by $LUC(G)$ the space of all bounded left uniformly continuous functions on G and by $C_0(G)$ the space of all

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