# Real interpolation method of martingale spaces

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**Abstract**

We describe the real interpolation spaces with a function parameter when we apply the real K-method of LionsPeetre to martingale Hardy spaces. As application we get interpolation spaces of the martingale Hardy-Lorentz spaces $\Lambda_q^s(\varphi)$.

**Keywords:** Martingale Hardy–Lorentz spaces, Lorentz spaces, Interpolation.
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## 1 Introduction

The family of martingale Hardy spaces is one of the important martingale function spaces. The study of the martingale Hardy spaces is extended to the martingale Hardy– Lorentz spaces [7, 4, 5].These spaces play an important role in the theory of Banach spaces since they have been defined are the objects of extensive investigations, results of which are contained among others in the papers [2, 6] and in probability theory and in statistics [3, 1]. Moreover, interpolation of martingale Hardy spaces is one of the main topics in martingale $H_p$ theory, and its theory has been applied to Fourier analysis. Here the interpolation spaces with a function parameter between martingale Hardy–Lorentz spaces are identified. Some results due to [8] are extended to interpolation with a function parameter.

## 2 preliminaries

To achieve our goal we first fix our notations and terminology. Let us denote the set of integers and the set of non–negative integers, by **Z** and **N**, respectively.

Let $(\Omega, \mathcal{F}, P)$ be a probability space. A filtration $(\mathcal{F}_n)_{n\in\mathbf{N}}$ is a non-decreasing sequence of sub-$\sigma$-algebras of $\mathcal{F}$ such that $\mathcal{F} = \sigma(\cup_{n\in\mathbf{N}}\mathcal{F}_n)$. We denote by $E$ and $E_n$ the expectation and the conditional expectation operators with respect to $(\mathcal{F}_n)_{n\in\mathbf{N}}$. For simplicity, we assume that $E_n f = 0$ if $n = 0$.

For a martingale $f = (f_n, n \in \mathbf{N})$ relative to $(\Omega, \mathcal{F}, P)$, denote the martingale differences by $d_n f := f_n - f_{n-1}$ with convention $d_0 f = 0$. The conditional square function of $f$ is defined by

$$s_m(f) := \left(\sum_{n\le m} E_{n-1} \mid d_n f \mid^2\right)^{1/2} \quad , \qquad s(f) := \left(\sum_{n\in\mathbf{N}} E_{n-1} \mid d_n f \mid^2\right)^{1/2} .$$

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